PREDICTING REGIME CHANGES IN NIGERIAN STOCK MARKET RETURN SERIES

By

AMOS OLUFEMI ADESIYAN

Matric No.: 68379

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Abstract

Regime change is the tendency of the Stock Market Returns (SMRs) for global market to change their behaviour abruptly due to changes in financial regulations and policies. This behaviour has no exemption to emerging markets like Nigeria. In literature, Markov Chain Models (MCMs) have been used to capture the stylised behaviour in 2-state regime; namely low and high return states, which limit the forecasting ability of the stock returns. The aim of this work was to extend the 2-state MCMs to 3- and 4-states for an improved forecast performance.

The MCM was employed to specify the state transition probability, P, limiting distribution, λ, the expected returns, μ and the occupancy times, $M(n)$. The behaviour of the SMRs was classified into five scenarios comprising 2-state regime defined as low and high return regimes (scenario 1), 3-state regime based on Mean $\pm$ 1SD (Standard Deviation) classification (scenario 2), 3-state regime based on Quartiles (Q) classification (scenario 3), 4-state regime based on Mean $\pm$ 1SD classification (scenario 4) and 4-state regime based on Quartiles (Q) classification (scenario 5). Following the classification, scenario 2 and 3 were defined as low, medium and high returns while scenario 4 and 5 similarly were defined as strong-low, low, high and strong-high returns. Index and Price data from All Share Index Return (ASIR), Dangote Cement Return (DANGCEMR) and Guaranty Trust Bank Return (GTBR) covering the period, 3 January 2006 to 29 June 2018, were used.

The limiting distribution, $\lim_{n\to\infty} P^n = \lambda$, the expected return time, $\mu = \frac{1}{\lambda}$ and the occupancy time $M(n) = \sum_{r=0}^{n} P^r$ (for $n > 0$) were obtained. The limiting distribution in days obtained for ASIR, DANGCEMR and GTBR, for each scenario were 4, 4, 4 for scenario 1; 15, 7, 8 for scenario 2; 8, 6, 7 for scenario 3; 15, 6, 9 for scenario 4 and 15, 6, 9 for scenario 5, respectively. The identified expected return time for the transition in days were also obtained for ASIR, DANGCEMR and GTBR, for each scenario as: 2, 2; 3, 1; 2, 2 for scenario 1; 716, 1, 716; 10, 1, 10; 11, 1, 9 for scenario 2; 4, 2, 4; 4, 2, 4; 4,2,4 for scenario 3; 778, 2, 2, 778; 10, 2, 5, 10; 11, 2. 3, 9 for scenario 4 and 4, 4, 4, 4; 4, 2, 2, 4; 4, 5, 3, 4 for scenario 5. The limiting distribution of the MCM obtained for scenario 1 was lower to that of scenarios 2 to 5 as the returns will transit into steady-state at days above 6 as against 4 for scenario 1. Occupancy times obtained for scenarios 3 to 5 gave a lower time period, an indication of short occupancy time. The transition probabilities obtained for scenarios 2 to 5 identified the persistence in state returns.

The 2-state regime was successfully extended to 3- and 4- state regimes respectively. The increase in the limiting and expected return times in days for scenario 3 and scenario 4 is good for an investor as it allows more room for investment before return to equilibrium.

Keywords: All share index**,** Regime classification**,** Steady state probability, Stock returns, Transition probability

Word counts: 495

Dedication

Almighty God, the entire unfortunate around the world.

My Parents; Elder Philip Akinbola Adesiyan (Late) and Esther M. Adesiyan

My wife, Temitayo Adesiyan

And my children; Ayodapo and Ayodamope Adesiyan

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I appreciate you all. Remain Blessed

Amos Olufemi ADESIYAN

January, 2021

Certification

I certify that this work was carried out by **Amos Olufemi Adesiyan** in the Department of Statistics, University of Ibadan

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Supervisor

Professor J. O. Iyaniwura

B.Sc, M.Sc, Ph.D

Department of Statistics

University of Ibadan, Ibadan. Nigeria

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Supervisor

Professor O. E. Olubusoye

B.Sc (Ilorin), M.Sc., Ph.D.(Ibadan)

Department of Statistics

University of Ibadan, Ibadan. Nigeria

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Acronyms used in the thesis

ASI	All Share Index
ASIR	All Share Index Return
BMV	Base Market Value
CMV	Current Market Value
DANGCEMR	Dangote Cement Return
DTMC	Discrete Time Markov Chain
GARCH	Generalized Autoregressive Conditional Heteroscedastic
GDP	Gross Domestic Product
GTBR	Guaranty Trust Bank Returns
MC	Markov Chains
MCA	Markov Chain Analysis
MCMs	Markov Chain Models
MLE	Maximum Likelihood Estimator
MSM	Markov Switching Model
NSE	Nigerian Stock Exchange
SMRs	Stock Market Returns
TM	Transition Matrix
TP	Transition Probability

CHAPTER ONE

INTRODUCTION

1.1 Background to the study

An equity market, which is generally referred to as stock market, is a public market that is characterized by the aggregation of traders (buyers and sellers) of tradable stocks of listed or quoted companies. Generally, the stock market plays an important role in the growth of the economy through equity issuances. More specifically, as a leading emerging market, Nigeria stock market plays a significant role as an important driver of growth in Nigeria, as well as in Africa. Equity issuances serve as major sources of finances for companies and government projects in Nigeria. It is therefore generally believed that a rising stock market would most likely influence an upward move of the country's economy. Basically, the stock market is often considered the primary indicator of the country's economic strength and growth. A positive change/movement of the stock index and share price is usually related to a similar positive change in business investment, and vice visa; which directly, or indirectly, affects the consumption of goods and services in the country. In a nutshell, stock market indices and prices are the instruments that measure the performance of the entire economy and the market. Consequently, investors around the world have always been interested in tracking the movement of various stock market indices and prices, in order to have an idea of how the global market behaves. However, the choice and decision of the most appropriate stocks to purchase usually determines the state of return on investment, made by individuals and/or corporate investors.

In addition, the decision to rightly select a positive return portfolio in the stock market largely depends on how adequately informed the investor or trader is, with respect to stock return analysis. A lot of studies have come up with statistical models and analyses, amongst which are; moving averages, regression analysis, time series analysis, Markov chain model, hidden Markov process, weighted Markov chain, etc., (Madhav, 2017). The investor's interest in the stock market is a global issue, and similarly, a lot of research had been conducted on the global level.

These studies include Jordan (1998) on London Stock Exchange; Sagaran (2011) on Kuala Lumpur Stock Exchange; Bialkowski (2004) on Stock Indices for Western and Central European Stock Exchanges; Bilgin (2005) on Istanbul Stock Exchange (ISE) 100 Indices; Chu et al. (2004) on monthly New York Exchange (NYSE) Index return; and Rey et al. (2014) on seven indices from global market, namely; CAC 45, DAX, FTSE 100, NIKKEI 225, NASDAQ, S & P 500 and Dow Jones. At the local exchange market, amongst others, Nigeria stock market return related research have been conducted by some notable researchers: Eseoghene, 2011; Idolor, 2009; Obodos, 2005; Yaya et al., 2013; Shittu et al., 2009; Tumala and Yaya, 2015; Oludoyi, 2003; Olakojo and Ajide, 2010; Osamwonyi and Asein, 2012; and Olubusoye and Olusoji, 2017.

All transactions usually pass through a stock exchange, which is oftentimes domiciled within each country and are so-named. These include; Nigerian Stock Exchange (NSE), Ghana Stock Exchange (GSE), Johannesburg Stock Exchange (JSE), and among others around world; where stockbrokers, traders and speculators can buy and sell shares (small and big companies), bonds (companies, federal, states and local governments), and also some other forms of securities. The size of any stock exchange market could be measured by the number of participants, in terms of number of quoted companies and/or market capitalization. The Nigerian Stock Exchange has a market capitalization of about N14.288 trillion, as at July 2019, with about one hundred and sixty (169) quoted companies. The market participants may include any of the following; individual retail (local and foreign) investors, local and foreign institutional investors like banks, insurance companies, mutual funds managers, trustees and pension fund managers, among others. The operations and transactions of the stock exchange market in Nigeria are regulated by the Nigerian Stock Exchange, as a self-regulatory organization (SRO); with the Securities & Exchange Commission (SEC), as an apex body that regulate the entire capital market activities in Nigeria, and also administer the Investment and Securities Act of 2007.

A brief note on the history of the Nigerian Stock Exchange, the All Share Index, which is one of the metrics that was literally attached to the growth of the stock exchange and similar exchanges in the world, is herein necessary. Also, a summary on Dangote Cement Plc and Guaranty Trust Bank Plc, as part of this research, would not be out of place. NSE was established in 1960 and was originally known as the Lagos Stock Exchange. However, in 1977, its name was changed to the Nigeria Stock Exchange. All the listed stocks formed the Nigeria Stock Exchange All Share

Index. Moreover, in terms of capitalization, the Nigerian Stock Exchange ranked the third largest stock exchange in Africa.

The Nigerian Stock Exchange is servicing the largest economy in Africa and simultaneously championing the development of Africa's financial markets competitiveness with the global market. With over 249 listed securities, the exchange offers one of the most advanced multi-product trading platforms in Africa. The All-Share-Index, of the Nigerian Stock Exchange, was formulated in January 1984 (January 3, 1984 = 100). Only the ordinary shares, often refer to as equities, are included in the computation of the index. The index is value-weighted and is computed daily. In the history of the Nigerian Stock Exchange, the highest value of the index (66,371.20) occurred on March 3, 2008. Apart from All-Share-Index (ASI), the Exchange had introduced the NSE-30 Index, a sample-based capitalization-weighted index, and also, five other sectorial indices were introduced to complement existing indices, namely; NSE Banking Index, NSE-Consumer Goods Index, NSE Insurance Index and NSE-Oil/Gas Index.

Dangote Cement Plc (Dangcem) is a multinational cement manufacturing company that is into manufacture, preparation, import, packaging and distribution of cements, and its related products. Dangcem is Nigeria's largest cement producer with three plants in Nigeria. The company's plant located in Obajana, Kogi state, is the largest in Sub-Saharan Africa, with 10.25mtpa across the three lines. Guaranty Trust Bank Plc, also tagged GTB or GTBank, is a quoted company in the financial sector of the Nigeria Stock Exchange. The bank was established as a limited liability company, with the license to provide the Nigerian public with business and other retail banking services in 1990, and started operations fully in February 1991. In September 1996, the bank approached the public, through the stock market, with her first initial public offers and became a publicly quoted company. In 2004, the bank undertook its second public share offering and raised over N11 billion from Nigerian Investors, through the capital market, to expand her operations. GTBank remains one of the most capitalized companies in the financial sector of the Nigeria Stock Exchange.

1.2 Motivation for the study

There have been a lot of studies on share price movements in Nigeria Stock Markets (NSMs), in the area of volatility of prices and returns, such as Yaya (2013), Davou et al. (2013), Afolabi and Dada, (2014), among others. To the best of my knowledge, many studies focused on the volatility of the returns, while a few studies on the probability and conditional probability of share movements focused more on the first-tier market of the Exchange. Some of the recent studies have examined the daily closing share prices of the quoted companies, using transition probabilities of the Markov Chains. One of such studies was carried out by Eseoghene (2011), which concluded that share prices in the Nigeria Stock Market were stable, while some other studies suggested instability of the stock prices (Christian and Timothy, 2014). All these studies have focused on stock prices, however, a departure from existing literature informed the focus of this present study. The market returns are examined using conditional probability theory from Markov Chain Models (MCMs).

Further motivation for this study is therefore not far from the eagerness to investigate, adequately, if there exist any evidence of state transitions in the capital market return series, and critically assess the main characteristics of each of the states. Financial markets are generally perceived to have a cyclical pattern and are best captured with regime-switching models. The nature of economic and financial variables causes the fundamental changes in behaviour, as a result of one or a combinations of the following; wars, financial panics, changes in government policies, changes and unstable fiscal policy and changes in market structure/market sentiments. Financial markets in most developing countries have experienced significant changes in government policies and capital market reforms. These changes may have led to changes in their return-generating processes. This study will therefore provide an alternative approach in determining the pattern of fluctuations in the stock market index.

1.3 Justification for the study

For investors and relevant stakeholders to be adequately guided in their choice of future investments and making strong investment decisions, more relevant studies relating to the analyses of stock markets characteristics are needed. Recently, attention has been drawn to the determination of the extent of volatility inherent in a financial series. This thesis focuses on the persistence of the three and four possible states of the market return in probability terms. The decision to use the MCMs process to study and predict market index returns and price behaviour was based on the random walk behaviour that characterizes stock prices, as noted by Bachellier (1914) and Fama (1965).

1.4 Statement of the problem

It is very necessary, to be on record that the Nigerian Stock Markets (NSMs) have contributed immensely to the country's Gross Domestic Product (GDP) and also, that share price movements have attracted both foreign and local investors, who are the major players, into the market. These feats have also contributed significantly to the credit rating of the country, through both local and international credit rating agencies. In the light of these, this thesis investigates the movement of returns in the NSMs using MCMs, and predicts, in probability terms, the 3-state and 4-state returns, as against the traditional 2-state returns, for relevant stakeholders, amongst which are investors (both local and foreign), traders, fund managers, academia and researchers.

Based on the problem identified, this research builds upon the work of Okonta et al. (2017), on the Makovian Analysis of the NSMs Weekly Index. The research focused on applying MCMs to the weekly returns, with the state returns classified into two states - Negative (Lower) and Positive (Upper). A careful review of existing studies, on the subject, revealed the following gaps:

1. Limitation of the classification of the states of market returns to two (upper and lower returns only), which is considered too restrictive.
2. The use of weekly data that resulted in the failure to capture immediate market noise/information, which are likely to affect market returns, either negatively or positively.
3. Failure to identify and highlight the inherent persistence and other plausible states of market returns.

1.5 Aim and Objectives of the study.

Following from the above listed identified gaps, the main aim of this work is to introduce two thresholds, for the classification of states in the MCMs framework, for daily stock market return series using 2-state, 3-state and 4-state discrete parameter Markov chain models. This extends the work of Okonta et al (2017). Basically, the objectives of the study are to:

1. Estimate the transition probability of daily stock market returns for the index and two most capitalized companies that are quoted on the NSE, using Markov model framework.
2. Obtain the stationary/limiting distribution of daily stock market returns movement for the index and the two quoted companies.
3. Obtain the expected transition recurrent time for daily stock market returns, based on the constructed Markov Chain model.
4. Obtain the occupancy times, the expected time spent at the various states of Discrete Time Markov Chain.

1.6 Research Questions

In line with the aim and objectives of this study, there are some pertinent questions that this research attempts to answer and they are as follows;

1. What are the different states likely to identify for returns' regime changes outside the traditional 2-state regime?
2. What will be the forecast, in probability terms, of the returns of the stock market, whose future value is influenced only by its current state, and not any prior activity that may lead the return to its current position?
3. What will be the probability distribution of states that remained unchanged, after various stages of transition, as time progressed, and similarly, what will be the stable probabilities for each of the state of market returns?
4. At what time will market returns for each of the regimes reach a stable point?

5. What is the expected amount of time that each of the regimes spends in a given state, during a given interval of time, whenever it re-occurs?

1.7 Organisation of the Thesis

This thesis contains five chapters, which are organized as follows: Chapter one provides a general introduction along with some background information. It further highlights the problems, the justification for the study, the identified knowledge gaps, as well as the aim and objectives with the addition of research questions. Chapter two presents a review of the literature on works relating to Markov chain models and its application to financial series. Chapter three discusses the adopted methodology. Chapter four presents the results, with interpretations and discussions, while chapter five gives the summary and conclusion, and also recommends areas for further research on the transition of market returns.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This chapter starts with the happenings in the global stock markets, with review of relevant literature on the various studies on the returns amongst the global (developed and emerging) markets. It continues with similar studies on the frontier markets, of which the Nigeria market belongs and this is followed by the theoretical and methodological reviews of stock market returns. The chapter ends with review of literature on the regime changing models.

2.2 Review of related global market studies

Global stock markets are the providers of financial services and products to international corporate organisations, governments and institutions across the globe. To achieve business consistency and long-term performance, global stock markets focused on building partnerships with local corporate, government and institutional clients. There are sixteen (16) stock exchanges in the world that each has market capitalization above $1 trillion. These exchanges are often referred to as the "$1 trillion club".

In accessing the ranking and status of stock exchanges, the movement of stock market indices plays a major role. It is worth noting that stock market indices are performance indicators for stock markets, and even act as barometers that inform us of the performance of the entire market, and consequently, the economy. Major players of the markets around the world keep track of the movements of different stock market indices, just to have detailed information on how the global markets behave. Market players not only track the movement of market indices, they have also shown deep interest in understanding how to adequately predict the market trend through the market indices. Table 2.1 shows the compilation of various global market indices.

Table 2.1: List of Global Stock Market Indices

	Global Market Indices	Domiciliary Stock Markets
1	CAC 40	French stock market index, a capitalization-weighted indicator of the 40 most important values among the 100 largest market caps on the Paris Stock Exchange (now Euronext Paris)
2	CSI 300 Index	A capitalization-weighted index of stocks intended to replicate the performance of 300 stocks traded in the Shanghai and Shenzhen stock exchanges
3	DAX 30	A list of the 30 big German companies traded on the Frankfurt Stock Exchange
4	Dow Jones Industrial Average	(DJIA or Dow 30) Big index of the US stock market. The average number of companies in the United States is 30 largest and most widely held
5	FT 30	An index based on 30 British companies' share prices
6	FTSE 100 index	A share index of the 100 UK companies with the highest capital, listed on the London Stock Exchange
7	Hang Seng Index (HSI)	A free float-adjusted stock market-weighted index in Hong Kong
8	Korea Composite Stock Price Index (KOSPI)	The list of all common stocks exchanged in the Korea Stock Exchange Market Division
9	Madrid Stock Exchange General Index (IGBM)	The main index for the Bolsa de Madrid (Madrid Stock Exchange)
10	NASDAQ-100	A stock market index of 100 of the largest foreign and domestic companies listed on the NASDAQ stock exchange.

11	Nikkei 225	Tokyo Stock Exchange's stock market index.
12	S&P 100	US stock market ranking of 100 leading stocks in the United State
13	S&P 500	Major US stock market index, a value-weighted price index of 500 large cap common stocks, traded actively in the US.
14	S&P CNX Nifty	The leading index on India's National Stock Exchange of for large companies
15	Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX)	A stock market benchmark of Taiwan Stock Exchange listed companies

Compiled by the Author

In exploring that Markov chain has no post-effect and also, to establish the appropriateness of the Markov Chain model to analyze and predict the index of stock market and closing price of stocks, in a more effective method - the market mechanism, Zhang and Zhang (2009) implemented MCMs to forecast the stock market trend in China. With this application of MCM in the stock market, the study achieved a relatively good result, and subsequently, recommended that the model could be used in other fields like Futures Market, Bond Market and Commodity Market. The authors also suggested that the results obtained from their Markov chain model for prediction could be combined with the other factors that significantly influence stock market variations, and that the method could be used as a basis for decision making.

On further applications of MCM, Mettle et al. (2014**)** used it with finite states to analyze the share price changes for five randomly selected equities on the Ghana Stock Exchange. This particular study concluded that the application of Markov chain model, as a stochastic analysis method in equity price studies, improved portfolio decisions. Consequently, the authors suggested the application of the Markov chain model as a tool for improving the stock trading decisions. The application of this method in stock analysis improved both investor's knowledge and chances of higher returns. Otieno et al. (2015) applied Markov chain model to forecast stock market trend of Safaricom shares, in Nairobi Securities Exchange, in Kenya. The Markov chain model was employed based on probability transition matrix and initial state vector to predict the Safaricom share price, using the data spanning a period from April 1, 2008 to April 30, 2012. In the study, Markov chain prediction was applied for the specific purpose of forecasting the probability of certain states of share prices, in future rather than absolute state. Using Markov chain model, the authors were able to predict the probability of each states of the shares of Safaricom.

In the forecast of Nepal Stock Exchange (NEPSE) Index, Madhav (2017) applied the Markov chain model and forecasted the future states, based on the randomness feature of NEPSE Index. The study aimed to explore the long run behaviour of NEPSE Index and the expected number of visits to a particular state, using the NEPSE Index of 2,741 trading days, starting from August 15, 2007 to June 18, 2017. The NEPSE Index showed three different states - increase, remain same and decrease.

An empirical study on stock index trend prediction, using Markov chain analysis, was also carried out by Vasanthi S, Subha V and Nambi (2011). In their study, the authors used first order Markov Chain model to predict the daily trend of various global stock indices, and compared it with that of extant traditional forecasting methods. Their results revealed that the Markov model outperformed the extant traditional models used in their study.

Moving away from the financial markets, Zhu et al (2012) used vegetable price fluctuation in China to establish the prediction Markov chain model. The notion for using Markov Chain to forecast market returns behaviour is necessary since stakeholders (investors) in the market are very interested in the direction of the market (trend), even before the takeoff of any investment decision.

From the list of global market indices mentioned in Table 2.1, Rey et al. (2014) considered seven indices, which covered a period of 21 years, namely; CAC 45, Dow Jones, DAX, Nasdaq, Nikkei 225, FTSE 100 and S&P 500, for the detection of stock returns abnormal states. They found out the possibility of three possible states as follows; a state of high-returns, low- returns, and high volatility with intermediate state of low volatility. Furthermore on the Markov Chain methodology, Bilgin (2005) applied the methods in testing the possibilities of Istanbul Stock Exchange's daily returns (ISE 100) indices following a process that is martingale (random walk). However, the result revealed that at any point in time, stock prices were a reflection of the available historical information.

Also for the application of normal distribution's Market switching mixture, Bialkowski (2004) studied the monthly returns of major stock indices, using Central Europe's emerging financial markets for modeling returns from Western and Central European Stock Exchanges on stock indices. In performing a robust test of the Stock Markets' January effect, Chu et al. (2004) used MSM, with monthly returns of the New York Stock Exchange NYSE equally weighted (EWR) index data, between January 1962 and December 1992.

2.3 Review of local market returns studies

Nigerian Stock market has continued to remain relevant as platform for securities exchange, both locally and internationally. The NSE All Share Index (NSE ASI) reached a 10-year peak of 45,092.82 in January 2019, as reported by The CEO of the Exchange (Oscar. N Onyema). This was as a result of the exchange being the 3rd best performer in the global market, according to CNN reports in 2017, and her positive performance, as it was reflected in the NSE ASI of 2017. In the past, a significant number of studies were carried out on the Nigeria stock market. The findings of these studies are to be discussed.

Starting the review on the local stock market, Choji et al. (2013) used MCM to predict the possible states, by illustrating the performance of the top two banks; the likes of Guaranty Trust Bank and First Bank of Nigeria. Data, spanning six years, was used to obtain the transition probability matrix, power of the transmission matrix and probability vector. They obtained the long run prediction of the share prices of these banks; whether the banks' share prices appreciated or depreciated or remained unchanged. They also estimated the probability of transition between the states, by taking the performance of the two banks together. In another study analyzing the price trends on the Nigeria Stock Market, Agwuegbo et al. (2010) used MCM in obtaining market transition probabilities between various states. A study similar to this was carried out by Doubleday et al. (2011).

In another strand of study relating to the phase of market returns, Yaya (2013), in his study titled, "Nigeria Stock Index, A search for Optimal GARCH model using High frequency data", attempted to fit the best Generalized Autoregressive Conditional Heteroscedastic (GARCH) model for All Share Index of the Nigeria Stock Exchange (NSE) returns. The research was based on modeling the volatility of returns and identifying the best model. Further work was carried out on estimates of the Bull and Bear parameters using a smooth threshold parameter nonlinear market model. On this, Yaya et al. (2013) did a comparative study between Nigeria and foreign stock markets. Their study was based on the study of phases of financial markets, using estimates of betas and nonlinearity in a smooth threshold parameter model. The model applied was an adaptation of the STAR model, which is mostly used in financial econometric modeling.

2.4 Review of Markov chain model theories

In literature, Markov chain models have been described as a statistical model that represent transition amongst successful outcomes of discrete time random variables. To buttress this point, the following literatures have understudied the Markov Chain models (Dynkia, 1965; Kemeny and Snell, 1976; Kijima, 1997; and Berchtold, 1999). The process is entirely visible because each observed outcome is exactly identified with one state of the process (Berchtold, 1999). The Markov Chain Analysis (MCA), upon which this thesis is hinged, is a prediction method that is based on a probability forecasting approach of Zhang and Zhang (2009). It has no after effect and it may be used effectively to analyze and predict the stock market index returns, as well as the closing stock price returns.

A very good number of researchers and scholars have tried to forecast stock market returns using different models as shown on Table 2.2. The models that are usually used are either linear or non-linear. This thesis therefore considers a non-linear model framework from another perspective, looking at the probability behaviours of market returns. The stock price behaviours have previously been widely explored in financial contexts, basically from the theory of random variables to the volatility in stock prices overtime. Historically, it was confirmed by Fama (1965) that stock prices satisfy the random walk hypothesis, and stated further that a series of price changes had no memory, which indicated that past price dynamics could not be used in forecasting future prices. From the theory of Efficient Market Hypothesis, changes in security prices can only be explained by the arrival of new information, which may be quite difficult to predict (Lendasse et al. 2008). Schwert (1989) used a model, where returns have either a high or a low variance, and in which a two state Markov process determined switches between these return distributions.

Turner et al. (1989) considered a univariate specification, with constant transition probabilities. They applied a markov switching model, where the mean or the variance, or both may differ between two regimes, using S and P monthly index data, for a period 1946-1989. It was shown that the Markov-Switching model provided a better statistical fit, to the data, than the ARCH models without switching parameters; by proposing a model with sudden discrete changes in the process, which govern volatility. This was established by Hamilton and Susmel (1993). Also, Filardo and Gordon (1998) specified a time varying transition probability model, where information contained in the leading indicator was used to forecast transition probabilities, and in turn, to calculate expected business cycle durations.

Drifill and Sola (1998) investigated whether there was an intrinsic bubble in stock prices, so that stock prices deviated from the values predicted by the present value model or deviated from the fundamental relationship between income and value. Rafiul and BaikunthuNath (2005), in their study used Hidden Markov Models (HMM) approach to forecast stock prices for interrelated markets. HMM was used for pattern recognition and classification problems because of its proven suitability for modeling dynamic system. HMM was a strong statistical tool as stated by the authors. Also, HMM was shown to handle new data, robustly, and was computationally efficient to develop and evaluate similar patterns. Yi-Fan et al. (2010) incorporated Markov chain concepts into fuzzy stochastic prediction of stock indexes, to achieve better precision and confidence. They comparatively examined the ANN and Markov models, and discovered that Markov model performed better, generated highly precise results and required only one input of data.

Table 2.2: Summary of Related Markov Chains application

Author & Year	Title	Contribution to Knowledge	Limitations	Intended Contribution
Eseoghere (2011)	The long-run prospect of stocks in Nigeria Capital Market; A Markovian Analysis	Used the Markov chain to analyze the long run prospect of stock prices in the stock market in the stock market	Used only 3 states and classification of state of price without use of threshold	Applying threshold-(1) use of mean and SD. (2) Use of Quartile range
Davou et al. (2013)	Markov Chain Model Application on Share Price Movement	Markov Chain Model application to model stock market share price movement.	Failed to look at the long-run distribution of the Markov Chain Model	Looking at the long-run limitation of the 2-states, 3-states and 4-states
Mettle et al. (2014)	A methodology for Stochastic analysis of share prices as Markov Chains with Finite States.	Used Markov with finite states for stochastic analysis of share prices. Established limiting distributions of the returns.	Like Eseoghene (2011), they did not employ the use of threshold in classifying the states	To apply threshold in state classification.
Jarsinthan et al. (2015)	A Markov Chain Model for Vegetable price Movement in Jaffina.	Use of 2 and 3 States as the threshold was applied through absolute median of the market daily changes	Failed to look at distribution in the long run and the states occupancy time.	Increase state to 4, use of mean and SD, use of Quartile
Maruf and Patrick (2016)	A three-State Markov Approach to Predicting Movement of Asset returns of a Nigeria Bank.	They employed a three state Markov approach to predict asset returns	No recourse to long run distribution and occupancy times of the returns	Occupancy times of the returns
Okonta et al. (2017)	A Markovian Analysis of the	Used of a two-state Markov	Failed to extend the state further	Extends to 3 and 4 states. Use of

	Nigerian Stock Market Weekly Index	Chain	to 3 and 4 states. Failed to use threshold for classification	threshold.

Compiled by the Author

2.5 Theoretical Review of Stock Market Returns

It is the primary objective of any investor to consider the possibility of earning good returns in any investment. Return, which is an amount realizable from investing in stocks, comes in the form of dividend, stock splits and the appreciation of capital. Both systematic and unsystematic risks are influenced by these returns; the former (systematic risk) comprising micro-economic variables, while the latter (unsystematic risk) includes factors that are specific to the individual company.

Moreso, returns on stock are very sensitive to a lot of factors; such as the country's political unrest, crises in the economy, natural calamities like fire and floods, wars and communities fight, movements in global prices for oil, inflationary effects, and government policy changes in both fiscal and monetary policies, and finally, regulatory agency policies.

From the theoretical background study, the following are identified as the major issues that have negative or positive impact on the stock market returns regime changes;

- Stock return Volatility
- Inflation
- Risk and Liquidity
- Interest rate
- Global Oil Price Movements/Shocks

Stock Return Volatility. This has been a focal point of discuss for a lot of scholars in areas of Economics and Finance, with lots of researches cutting across areas of volatility. The effects of speculative trading on stock return volatility was investigated by Hsin and Luo (2003), and found such speculative trade to have a significant positive impact on stock return volatility. It therefore means that the impact of speculative and non-speculative stocks could in a way determine the path of stock market regime changes.

Inflation. It has been a known significant factor that affects all economies around the world and in fact, was an interesting area to a lot of research analysts and scholars, globally. A significant number of researchers took special interest in analysing the relationship or rather the inflation impacts on stock markets or the realized returns from the markets, vis a vis the changes in the regimes of stock markets. From various scholarly reviews; Alagidede and Panogiotidis (2012) studied the correlation between returns on stock and inflation for the G7 countries, and discovered a strong relationship for United Kingdom and Italy. Similarly, estimation of long-term trend of stock-related inflation was carried out by Boucher (2006). Also established was the relationship between inflation and returns on stock, which was perceived to be influenced by monetary policy, and demand and supply uncertainties (see Du.D, 2006). Further studies on inflation, as related to stock returns, was the framework for committee on monetary policy (MPC) implication for the monetary policy returns on stock relationship in the United Kingdom, as was studied by Chortareas and Noikokyris (2014).

Interest Rate. Theory has established that correlation exists between changes in interest rate and stock market. Intuitively, an increase in interest rate results in a drop in the activities of the stock market of a country, according to Ali (2014). The reason being that if potential investors are opportuned to get higher returns on investments, such as treasury bills, this will lead to reduction of activities in the stock market, which would, consequently and most likely, affect stock returns. However, various studies, testing of the influence of interest rates on frontier and emerging economies' stocks markets, mixed results have been displayed. This is because the relationship between interest rates and returns on stock can differ over time, based on the business activities of the countries, as opined by Chen and Hu (2015).

Stock performance is usually calculated by list of stocks, while Treasury bill rate is measured by the interest rate. As earlier stated, most studies conducted in both emerging and developing economies showed a co-integrating relationship among stock markets and other factors. One of such studies has been Khan et al, (2012) relating to Kenya. The researcher used regression analysis to study a ten-year period of monthly data, and showed a poor correlation between stock performance and interest rates. A similar study, which showed no meaningful relationship between interest rate and share price, was carried out by Chirchir (2013). Using a shorter time period, Obura and Anyango (2015) found comparative results. From January 1998 to January 2014, for two emerging economies (Egypt and Tunisia) Barakat et al. (2015) performed a similar report. Their results showed evidence of co-integration and the causal relationship between the interest rate and the exchange rate, market index and money supply.

Risk and Liquidity also have some level of impact on the direction of the stock market returns, which also dictates the various phases of regime changes. In terms of liquidity volatility, market returns and liquidity relationship were analysed by Chang et al. (2010), using Tokyo Stock Exchange (TSE) data. Results showed negative liquidity and returns on stock relationship. Another scholar, Chen and Hill (2013), established a robust and effective relationship between the liquidity and returns on stock. In the process of examining the returns on stock and risk correlation, Xing and Howe (2003) came up with different market factors, which could be considered. Henriques and Sadorsky(2001) studied multifactor-related risks and reports of significant effects on the return of stocks. The liquidity behaviour in emerging markets was also documented by Jun et al. (2003). However after a critical evaluation, a significant positive relationship between returns on stock and liquidity was found by the authors. Nevertheless, in emerging markets, little work has been done in the field of stock returns liquidity and risk.

Global Oil Price Movements. The last one and half decades have been characterized by a greater oil price fluctuations. However, a lot of research analysts showed interest in analysing the relationship between returns on stock and price movements of oil; although, more work is still required to be done on this. Evidence of significant relationship between oil price movements and economic policy influencing stock returns was revealed by Kang & Ratti (2013). For European stock markets, Cunado and de Gracia (2014) ascertained that oil price variations had significant and negative impact on stock market returns in most of the states. Gupta and Modise (2013) used the VAR model to evaluate the relationship between oil shocks and stock returns. However, coming back home to frontier markets like Nigeria, Fowowe (2013) reported insignificant association between oil prices and stock returns of the Nigerian Stock Exchange. Works done by Mohanty et al. (2010) and also Chatrath et al. (2014) could be considered as part of other notable researches in this area.

Other research areas relating to returns of the stock market over the years as show in Table 2.3 are (i) sentiments and effects of over-reaction on stock return; (ii) return on equity and leverage; (iii) mutual funds and returns on stock; (iv) monetary policy and returns impact; (v) the effect of terrorism on stock returns; (vi) political systems and returns on stock (vii) business cycles and returns on stock; (viii) up-to-date activity and returns on stock; and (ix) bargain hunting, capital appreciations, liquidity and returns on stock.

Table 2.3: Previous related theoretical reviews on Stock Market Returns

Author(s) & Date	Title	Theory effects	Findings
Alagidede, P., and Panagiotidis, T. (2012).	Stock Returns and Inflation: Evidence from quantiled regressions	Inflation	The study found positive relationship for countries such as UK and Italy
Ali, H. (2014).	Impact of interest rate on stock Market; Evidence from Pakistani market	Interest rate	The study found interest rate to have negative impact on the stock market
Boucher, C. (2006)	Stock prices: inflation puzzle and the predictability of stock market returns	Inflation	Estimation for a perspective of long-term inflation patters with stock returns.
Chang, Y. Y., Faff, R., and Hwang, C. Y. (2010).	Liquidity and stock returns in Japan: new evidence	Liquidity	The study revealed negative relationship between liquidity and stock returns
Chen, H., & Hu, D. (2015).	The Interaction between Interest Rates and Stock Returns: A Comparison between China and US.	Interest rate	Depending on the business activities of the countries, the relationship between interest rates and returns on stock may vary over time
Chen, J., & Hill, P. (2013).	The impact of diverse measures of default risk in UK stock returns	Risk and Liquidity	The study established a consistent and stable relationship between liquidity and returns on stock.
Chirchir, D. (2013).	The relationship between share prices and interest rates: evidence from Kenya.	Interest rate	No significant causal link was found between interest rate and share price was established.
Chortareas, G., and Noikokyris, E. (2014).	Monetary policy and stock returns under the MPC and inflation targeting	Monetary policy and Inflation	Effects of the monetary policy committee (MPC) system for financial returns relationship in the United Kingdom
Cunado, J.; De Gracia, F.P. (2014).	Oil price shocks and stock market returns: Evidence for some European countries	Oil Price	The study showed that oil price variations have significant and negative impact on stock market returns

Du, D. (2006).	Monetary policy, stock returns and Inflation	Inflation	The combination of inflation and return on stocks as influenced by monetary policy and the risks of supply and demand
Fowowe, B. (2013).	Jump dynamics in the relationship between oil prices and the stock market: Evidence from Nigeria	Oil price	There was an insignificant association among oil prices and stock returns on the Nigerian Stock Exchange
Hsin, C. W., Guo, W. C., Tseng, S. S., and Luo, W. C. (2003)	The impact of speculative trading on stock return volatility: the evidence from Taiwan	Speculative Trading and Volatility	The study established that speculative trade had a significant positive impact on stock return volatility.
Kang, W., and Ratti, R. A. (2013).	Oil shocks, policy uncertainty and stock market return	Global Oil prices	The interrelationship between price of oil movements and the economic blueprint that affects the return on stocks was noted.
Khan, Z., Khan, S., Rukh, L., Imdadullah, K., and Rehman, W. (2012).	Impact of Interest Rate, Exchange Rate and Inflation on Stock Returns of KSE 100 Index	Interest rate, Exchange rate and Inflation	The poor interaction between the stock returns and the interest rates was highlighted.

Compiled by the Author

2.6 Methodological Review of Stock Market Returns

Better understanding of the stock market returns revealed that it follows a random walk; however future returns are hard to predict or forecast. There are several reasons to try to predict the prices of the stock market. Financial gain is the most fundamental of these. Any system in competitive market that can reliably select winners and losers, will make the system's owner very wealthy. Most people, including analysts, investment practitioners and ordinary investors, are therefore constantly searching for a better system, which is going to produce high returns for them. The stock market forecast is certainly a fascinating assignment. There are a quite number of methods available in the literatures that have been used to perform this task, which include the following; Efficient Market Hypothesis (EMH), Capital Asset Pricing Model (CAPM), Arbitrage Pricing Theory, Time Series Analysis and Markov Chain Process.

2.6.1 The Hypothesis of an Efficient Market:

The Efficient Market Hypothesis (EMH) notes that all known share information are fully captured by the price of the stock at any time. Due to the optimal use of all known information by market participants, price variations are random as new information occurs randomly. Share prices therefore follow a "random walk" process, such that an individual investor cannot beat the market. Given this very strong statement, which in fact tends to be false, there were inconclusive evidence(s) to deny the EMH. Extant studies have either accepted or rejected the EMH, depending on the peculiarity of the market events. A lot of this research has used neural networks to support their arguments. However, since the neural network is just as good as it has been trained to be, it may be difficult to argue that the hypothesis based solely on performance of the neural network is accepted or rejected. In reality, the stock market crashes, such as October 1987 crash in the market which invalidates the EMH because it is not on the basis of random information, but arises as a result of alarming investor worries. The EMH is however, essential because it runs counter to all other methods of analysis. If the market cannot be beaten, then analyzing the technical, fundamental or time series should not lead to better result than a random calculation. This indicates that the EMH may not be true in reality, that many market stakeholders were willing to constantly outperform the market. In an ideal world, the EMH could be valid with equal distribution of information, but today's markets have a good number of privileges.

2.6.2 Chaos Theory

The Chaos theory is a relatively new approach to modelling nonlinear dynamic systems, such as the stock market. With the assumption part of the process is random and that part of the process is deterministic, Chaos theory analyses a process. Chaos is a nonlinear, seemingly random system. To check whether the system is chaotic (has chaos in its time series), numerous theoretical tests have been established. The chaos theory is an attempt to demonstrate that there is an apparent randomness. This argues that the stock market is volatile and unpredictable; the concept of chaos opposes the EMH. Basically, it's a chaotic system which is a mixture of both deterministic and random process. Using regression fitting, the deterministic process can be characterized, while the random process can be characterized by distribution function's statistical specifications. Thus, the nature of a chaotic system will not be fully captured using only deterministic or statistical techniques. The capability of neural networks for capturing deterministic and random features make it ideal for chaotic systems modeling.

2.6.3 Capital Asset Pricing Model

Capital Asset Pricing Model (CAPM), which is based on Markowitz Portfolio Theory, describes a portfolio's risk-return relationship. CAPM became very popular and widely used in many empirical studies because of the simplicity of the theory structure. The simplicity of the application structure was, however, criticized by economists. Amongst the criticisms are Breeden (1979), who argued that the CAPM theory was based on relaxed assumptions, and he developed expended CAPM to forecast stock returns. Similarly, Lewellen (2000) also claimed that CAPM does not describe, fully, the behaviour of stock returns.

2.6.4 Arbitrage Pricing Theory (APT)

This method of forecasting returns was introduced by Ross (1976). The main basic assumption of APT is based on the absence of arbitrage in the market. However, these returns can be calculated if there is no arbitrage opportunity. Capital markets are known to be perfectly competitive and basically, the fact remains that investors always prefers more wealth to less wealth. APT is less restrictive than CAPM in its assumptions. It is on this note that there is only one factor in CAPM but in APT there are n factors, which affect the expected rate of return.

2.6.5 Forecasting with Time Series Analysis

The traditional prediction of the time series analyses uses historical data and attempts to approximate future values of a time series, as a linear combination of historical data. There are two basic forms of time series prediction in time series econometrics: univariate (simple regression) and multivariate (multivariate regression) (see Maddala, 1992). The most common tools used in econometrics, for predicting time series, are these types of regression models. The way they are applied in practice is that the series under prediction is formed first of all by a number of factors that influence (or influence is believed to be more specific) it. These factors are the model's explanatory variables x_{it}. A mapping is then performed in between their values x_{it} and the time series value y_t (y is the parameter to be explained), to construct pairs $\{x_{it}, y_t\}$. Both pairs are used in the formulation of the variable to be clarified, to describe the value of each explanatory variable. In other terms, a linear combination of x_i, which approximates y optimally, is defined. Univariate modeling is based on a single explanatory variable, and multivariate models are based on more than one variable. Models of regression were used for forecasting time series on the stock market. Pesaran and Timmermann's (1994) study is a good example of using multivariate regression.

2.6.6 Markov Chain Process

Markov process, also referred to as random or stochastic process, is applicable in taking decision in situations, where the transition probability of a future state depends on the current state, irrespective of the process in which the occurring state was attained. The idea of the Markov chain was first introduced by Andrei Andreyevich Markov (1856-1922), a Russian Mathematician in 1906. His study involved studies of the behaviour at the beginning of a system, with the intention to predict the behaviour in the near future of the same system. In a subsequent paper published in 1913, Markov applied the chains methodology to the distribution of vowels and consonants in A.S Pushkin's poem Eugeny Onegin. General notion about the Markov process was advanced by A.N. Kolmogrov, W.Feller and a whole lot of other researchers. Over time, more areas of applications of Markov chains have been revealed (Basharin et al., 2004). This discovery cut across various subjects of study, namely; Agricultural Science, Botany, Criminology, Demography, Educations, Engineering, Economics and Finance, Industry, Medicine, Meteorology, Political Science, Psychology, Sociology, Sports and Games, Veterinary Science and Zoology. In recent times, economists have recognized its importance, as an economic analysis tool that is generally accepted and often applied in econometrics.

The theory of Markov chain was developed during the early 20^{th} century by a Russian Mathematician named Andrei Andreyevich Markov. He was a Mathematics student under some famous Russian mathematicians, such as Aleksanah Korkin and Pafnuty Chebyshev. Markov advanced his knowledge, most especially, in the fields of algebra and probability theory, with his early works based mainly on number theory and analysis. His interest in the law of large numbers and its extensions led him to the development of the theory of Markov chains, a name coined after Markov himself. Markov chain process is therefore used in modeling technical problems and is very effective in modeling financial time series.

Prediction of stock market appears to be a difficult task. In the past, researchers have tried to develop series of methodologies that shareholders and brokers could make maximum possible gains. At the earliest, market players would use various methodologies, such as Time Series Analysis, Fundamental Analysis, and Technical Analysis. After this efficient market hypothesis, Chaos theory and some other stochastic process models have been used.

2.7 Regime Changing Models

Following the review of various methodologies, which address stock market returns, the inclusion of different theoretical polices, is bound to create regimes/phases in stock market returns. Regime-switching models are time-series models with parameters that may be used for various values, in each of the defined numbers of "phases". The stochastic method, believed to have triggered regime changes, is included as part of the model that opens configuration of model-based forecast which include the possibility of future regime changes. The regime is directly observable in operation at any point in time for every special situation. More precisely, the regime is not observed, and the analyst has to make an inference as to which regime the system has been in, at times past. The main use of these models for literatures in applied econometrics was to explain shifts in the complex performance of the macro-economic and economic time series.

Regime-shifting models can usefully be divided into two categories: the "threshold" and the "Markov-switching". The key distinction between the two methods is how the origin of the system phase is being modeled. Markov-switching models, have been applied to econometrics by Goldfeld and Quandt (1973), Cosslett and Lee (1985), and Hamilton (1989), and conclude that the system changes develop in line with the Markov chain. Regime-models of transition have become an extremely popular modeling tool for used work. Of particular note are regime-shifting models of economic output indicators, just like actual Gross Domestic Product (GDP), which has been used as a template to define the business cycle phases. Few of such models are Hamilton (1989), Beaudry and Koop (1993), Chauvet (1998), Pesaran and Potter (1997), Potter (1995), Tiao and Tsay (1994), Kim and Nelson (1999b, 1999c), Van Dijk and Franses (1999), Öcal and Osborne (2000), and Kim, Morley and Piger (2005). Amongst the list of other relevant studies which involves modeling on policy changes in interest rate cycles and inflation are as follows; Ang and Bekaert, (2002), Evans and Wachtel, (1993); Garcia and Perron,(1996); also for phases of high and low volatility in returns on stock; Turner, Startz and Nelson, (1989; Hamilton and Susmel, (1994); Hamilton and Lin, 1996; Dueker, (1997), changes on government blue prints on "rules" of the Federal Reserve; Kim (2004), Sims and Zha (2006), and time difference in the response of economic output to monetary policy actions by Garcia and Schaller (2002); Kaufmann,(2002); Lo and Piger, (2005),Ravn and Sola,(2004).

In conclusion, extant studies indicate that the operational status of the stock market is subject to the influence of various factors from the market, such as multiple market forces from both sides, the fundamental state of the stock itself, macroeconomic policy, trade and economic degrees and psychological factors of investors. Therefore, no single method can accurately predict changes in the stock market every day. Markov Chain analysis, which this study aims at exploring, is a prediction method based on probability forecasting approach with no after effect and may be effectively used to analyze and predict the stock market index and closing stock price under the above market mechanisms (Zhang and Zhang 2009).

CHAPTER THREE

METHODOLOGY

3.1 Introduction

In this chapter, the focus is on the adopted methodology of the study, which comprises of Data source and scope of study, Markov chain process (MCP), the classification and concept of MCP, Assumption of Markov chain models, computation of All Share Index and its returns, companies' price returns. The chapter elaborate on the Markov chain methods used to answer the research questions and also discussed the test of independence as follows; model specification, construction of the MCMs, transition probabilities and estimation of transition probabilities, n-step state transition probabilities, state of Markov chains, steady state, limiting distribution, mean return times and occupancy times .

3.2 Data Source and scope of the study

This study covered the daily All Share Index (ASI) of the Nigerian Stock Exchange, the daily share prices of Dangote Cement Plc, which is one of the largest capitalized industrial company listed on the Exchange and Guaranty Trust Bank amongst the leading high capitalized banking company. It captures the returns movement, as All Share Index Returns (ASIR); Dangote Cement Returns (DANGCEMR); and Guaranty Bank Plc Returns (GTBR).

In the study, the sample period of the all share index ranged from January 2006 to June 2018, while Dangote Cement daily price ranged from May 2010 to June 2018, and Guaranty Trust Bank daily price ranged from January 2006 to June 2018. These resulted in a total data points of about 3,066 trading days for the index and GTBR, and 1,867 trading days for DANGCEMR. The daily stock market index data was collected from the website of two capital market operators, namely; Cashcraft Asset Management Limited and Skyview Capital Limited; as well as the bulletins of the Nigerian Stock Exchange.

3.3 Markov Chain Process

In this study, the Markov chains model is applied to analyze and predict the various regimes/phases in financial market returns. These phases can be expressed by first-order discrete-time Markov chain model and Markov chain models of higher-order (second-order). Markov chain is a sequence of random variables that evolves over a period of time. It is a system that undergoes transition between states in the system, and this is characterized by the property that the future is independent of the past, given the present. This means that the next state in the Markov chain depends only the current state and not on the sequence of events that preceded it. This type of "memoryless property" of the past is referred to as the Makovian property.

The classification of Markovian systems can be grouped into four types, according to the time and state parameters, namely; discrete time and state, continuous state and discrete time, discrete state and continuous time, continuous time and state. The Markov process of discrete time and state, is often referred to as discrete time Markov chain (DTMC) and has wide applicability that cuts across various subjects, amongst which are Biology, Physics, Chemistry, Economics and Finance, Sports and Music. A DTMC is divided into two main categories; one that is time homogeneous (the case where those Markov chains exhibit the constant transition probabilities states) and the second, which is time-inhomogeneous (the case where the transition probabilities between the states are not constant but time dependent). Further classification of a DTMC could be according to the characteristics of the respective state-space. Like states can be reached from any other states, while many other states cannot leave once they have entered. In view of this, there exist different classes of DTMC, which are categorised as follows; Irreducible DTMC, Aperiodic DTMC and Absorbing DTMC. These classifications will be used in simplifying the analysis of the behaviours of the Markov chain system in long-term.

3.4 Classification and Concept of the Markov Processes

The Markov process can be categorised into four types, which is attached to the character of the values denoted by 't' and {ASIR}. Meaning that this classification of the Markov process is a function of the time parameter and its state space. Based on the state space, a Markov process can either be a discrete or continuous state Markov process. Similarly, with respect to time, a Markov process can either be a discrete-time Markov process or a continuous-time Markov process (Ibe, 2013).

Therefore, naming them proceeds as details in Table 3.1;

1. A discrete random sequence that meets the Markov chain property is termed, "Discrete Parameter Markov Chain", where both $'t'$ and $\{ASIR_t\}$ are discrete.
2. A continuous random sequence that meets the Markov chain property is termed, "Discrete Parameter Markov Process", where $'t'$ is discrete and $\{ASIR_t\}$ is continuous.
3. A discrete random process that meets the Markov chain property is termed, "Continuous Parameter Markov Chain", where $'t'$ is continuous and $\{ASIR_t\}$ is discrete.
4. A continuous random process that meets the Markov chain property is referred to as "Continuous Parameter Markov" process, where both $'t'$ and $\{ASIR_t\}$ are continuous, as contain in Table 3.1.

Table 3.1: Categories of Markov chain state.

	Time	State Space	Combinations
1	Discrete	Discrete	{Discrete, Discrete}
2	Discrete	Continuous	{Discrete, Continuous}
3	Continuous	Discrete	{Continuous, Discrete}
4	Continuous	Continuous	{Continuous, Continuous }

Source: Summarized by the Author.

According to Hallberg (1969), the developments in the study of many economic variables overtime can be attributed to stochastic processes. Markov chain can therefore be used to predict future developments of certain variables. Furthermore, for the analysis of structural changes in one sector, the estimation of Markov chains is an often used approach (Disney et al., 1988). In a Markov process, the movement of an investment returns from one specific period to another is represented by transition probabilities. This study is however based on the discrete time discrete-state process.

3.5 Assumptions of Markov Chain Models (MCMs)

In applying the Markov chain models, the following assumptions were taken into cognizance, for the smooth running of the model:

1. A fixed set of states
2. Fixed transition probabilities and the probability of getting from any state to another, through series of transition.
3. A Markov process converges to a unique distribution over states. This means that what happens in the long run will not depend on where the process started from or what happened along the way.
4. What happens in the long run will completely be determined by the transition probabilities, which is the likelihood of moving between the various states.

3.6 Definitions of some terms

Probability Vector: This refers to a matrix having only one row, and consists of non-negative entries, such that the sum of the entries in the row equals to one (1)

Regular Transition Matrix: This is a matrix referred to as regular if some matrix power consists of all positive entries.

Stochastic Processes: This is a mathematical model, in a probabilistic operation, that evolves overtime**.** A Stochastic process is a random variables $\{Z_{(t)} : t \in T\}$ family, where t generally indicates time. That is, a random number Z_t, which is observed at each moment time t in set T.

State Space (S): This is a stochastic process, which defines the set of all possible values that a random variable assumes (the set of actual values that $Z_{(t)}$ can take). The state space S is discrete if it is finite or countable. The state space S is the set of states that can be in the stochastic process.

Definition 3.1

$\{Z_{(t)} : t \in T\}$ is a discrete-time process if the set T is finite or countable. And in practice, this generally means $T = \{0,1,2,3,...\}$. However, a discrete-time process $\{Z(0), Z(1), Z(2), Z(3),...\}$ is a random number associated with every time 0, 1, 2, 3, …

Definition 3.2

$\{Z_{(t)} : t \in T\}$ is a continuous-time process if T not finite or countable, and is usually denoted as follows; $T = \left[0, \infty\right)$ or $T = \left[0, K\right]$ for some K. However, a continuous-time process $\{Z_{(t)} : t \in T\}$ has a random number $Z_{(t)}$, which is associated with every point in time.

3.7 Computation of All Share Index

Stock market index, basically referred to as stock index, is a measurement of the value of a group/section of the stock market. It is usually computed from the prices of a selected stock of interest, and can also be computed for the entire market. The Nigeria Stock Exchange tagged her stock market index as All Share Index (ASI).

In the process of detecting or measuring the extent and direction of the general price movement on the trading floor of the Nigerian Stock Exchange, the Exchange started to compute and publish Stock Exchange Index in January 1984. Summarily, index is an aggregate of the market capitalization of all the companies' equities listed and traded on the exchange.

The computation of the index is as follows:

NSE All Share is given by;

$$ASI = \frac{Current\ Market\ Value}{Base\ Market\ Value} \times 100 \qquad 3.1$$

$$= \frac{CMV}{BMV} \times 100 \qquad 3.2$$

$$= \frac{\sum_{i=1}^{n} P_{a_i} Q_{a_i}}{\sum_{i=1}^{n} P_{b_i} Q_{b_i}} \times 100 \qquad 3.3$$

where

P_a and Q_a represent the market price and quantity/number, respectively, of listed ordinary share at the current date; while P_b and Q_b are the market price and quantity/number, respectively, of listed ordinary share at the base date and $i = 1, 2, \ldots, n$ is the number of the listed companies that constitute the Index.

3.8 All Share Index Returns

Definition 3.3

Let $\{ASIR_t, t \in T\}$ be a Markov chain with index T and state space S. In this study in particular, if $S = \{1, 2, 3\}$, then $\{ASIR_t, t \in T\}$ is said to be a three- state Markov chain.

The computation of the return on the All Share Index Returns (ASIR) is as follows:

3.8.1 Index Returns

Let ASI_t be the index of the market at time t and let ASI_{t-1} be the index of the market at time $t-1$ on the trading floor of the Nigerian Stock Exchange. Also, the simple net return is denoted as $ASIR_t$ and given as follows:

$$ASIR_t = \frac{ASI_t - ASI_{t-1}}{ASI_{t-1}} \qquad 3.4$$

And this can be further brokedown to;

$$ASIR_t = \frac{ASI_t}{ASI_{t-1}} - 1 \qquad 3.5$$

And for the computation of the stock returns, the same procedure as in equation 3.4 is applied and taking P_t as the closing price of a company's share for day t on the trading floor of the Nigerian Stock Exchange. The computation of Guaranty Bank Plc and Dangote Cement Price returns will also follow the same procedure.

3.9 Descriptive Statistics

The descriptive statistics for the series in this study: the All share index, Dangote Cement returns and Guaranty Trust Bank returns, which includes the mean, standard deviation, quartiles, skewness and kurtosis, will be discussed in subsequent sections.

3.9.1 Mean

An arithmetic mean is defined by an algebraic formula that captures all observations. It is regarded as being representative of the given set of data and also widely used in advanced statistical analysis (further analysis and algebraic calculations often carried out using the mean). It can be computed even when the detail of the distribution is not known, but some of the observations are known. The mean is least affected by fluctuation of sampling and it is based on the value of every item in the observations.

3.9.2 Standard Deviation

It is based on all the observations of the data and shows the extent of how much the observations are clustered around a computed mean value. It also gives a more accurate knowledge of how data is distributed. Moreover, it is not really affected by extreme values and is considered as the best measure of variations.

3.9.3 Quartile

This is a statistical term, which divides or partitions a set of observations into four well defined and equal parts, based on the values of the data and their positions in the entire set of observations. The quartile measures the spread of values above and below the stated partition when dividing the distribution into four groups. As median divides the data set into two, so that 50% of the observations lie below the median and 50% of the observations lies above it, so also, the quartile breaks down the observations into quarters so that 25% of the observations are less than the lower quartile, 50% are less than the mean and 75% are less than the upper quartile. The quartiles, as applicable to this research, are three (3) - lower quartile, median and upper quartile, thus forming four parts of the data set. The lower quartile or first quartile is denoted by Q_1 and is the middle number that falls between the smallest value of the data set and the median, Q_2 is the second quartile (also referred to as the median). The upper or third quartile is denoted by Q_3 and is the central point, which lies between the median and the highest observation.

3.9.4 Skewness and Kurtosis

By definition, skewness refers to asymmetry (a lack of symmetry) in contrast to the symmetrical bell curve normal distribution of a set of data. Having a curve that is shifted to the left or to the right, such that no part looks like the mirror of the other, then the data set is said to be skewed. However, skewness can be quantified as a representation of the extent to which a distribution differs from normal distribution. A normal distribution has a skewness value of zero. A data set can be positively-skewed (or right-skewed) or negatively skewed (or left-skewed). The kurtosis, on the other hand, measures the heaviness of the tail (whether the data is heavy tailed or light-tailed), relative to the normal distribution. The kurtosis of a standard normal distribution is 3.

Following these definitions, equations 3.6 to 3.9 are the computation of the descriptive statistics for ASIR.

$$\mu_{ASIR} = \frac{1}{N}\sum_{n=1}^{N} ASIR_n \tag{3.6}$$

$$\sigma^2_{ASIR} = \frac{1}{N-1}\sum_{n=1}^{N}\left(ASIR_n - \mu_{ASIR}\right)^2 \tag{3.7}$$

$$S_k = \frac{1}{(N-1)\sigma^3_{ASIR}}\sum_{n=1}^{N}\left(ASIR_n - \mu_{ASIR}\right)^3 \tag{3.8}$$

$$K_t = \frac{1}{(N-1)\sigma^4_{ASIR}}\sum_{n=1}^{N}\left(ASIR_n - \mu_{ASIR}\right)^4$$
$$Excess\ Kurtosis = K_t - 3 \tag{3.9}$$

For the other two variables: DANGCEMR and GTBR, a similar calculation is followed. And in case of further definitions on this study, ASIR will be used as a generalized variable for GTBR and DANGCEMR in defining MCMs variables.

3.10 Deterministic and Stochastic Models

The word "Stochastic" comes from the Greek word, "Stokhazesthai", which means to aim at or guess at (Dobrow 2016). A stochastic process, which often refers to a random process, is one in which the outcomes are uncertain, a contrast with the deterministic system, where there is no randomness. In a deterministic system, output is always produced from a given input, where functions and differential equations are used to describe the deterministic process. Similarly, random variables and probability distributions are the input for stochastic system.

3.10.1 Stochastic Processes

A stochastic process is a family (or collection) of random variables that are indexed by a parameter, such as time (T). The major elements that distinguish between stochastic processes include the nature of its state space, its index set T, and the dependence relations among the random variables $\mathrm{ASIR(t)}$. Following Ross (1989) the possible values, which $\mathrm{ASIR(t)}$ can take, are called the states of $\mathrm{ASIR(t)}$. Changes in the values of a stochastic process $\mathrm{ASIR(t)}$ are called transition between the states of $\{\text{ASIR(t)}\}$. If $\text{ASIR(t)} = \text{i}$, then the process is said to be in state i at time t. Stochastic processes are characterized by three principal properties, namely; the state space, the parameter set and the dependence relations between the various random variables ASIR(t).

The state space is a collection of all possible values that the random variables can take. Put differently, it is the sample space of the random variables. If we let $ASIR_i \in [0,\infty]$ represent random variables for all i, the state space of the stochastic process is $[0,\infty]$. On the application of stochastic processes, in 1852 Brussels applied stochastic processes to model rainfall patterns, in 1845 Brussels also applied branching process invented to predict the chance that a family name goes extinct. In 1905, Einstein described the mathematical properties of the Brownian motion; while he used Poisson processes to describe radioactive decay in 1910 and in 1914. Birth and death process (a type of CTMC) was used to model epidemics.

3.11 Test of Independence

Stochastic variable sequences have Markovian properties, which is a necessary condition for the analysis of Markov chain model and for calculating the transition probabilities using χ^2. The methods process is as follows. Let n_{ij} represent the frequency of the process of a step from which the starting state i moves to the state j in a sequence $ASIR_1, ASIR_2, \ldots \text{ASIR}_n$. The current state and the next state are said to be independent, if the probability distribution of one state is not affected by the presence of the other state. Suppose that n_{ij} is the observed frequency of the events that occur from state i to state j, and e_{ij} is the corresponding expected frequency; the null hypothesis of the independence assumption is to be rejected, if the chi-square probability value is less than the specified significance level α. The computation defines as in equation 3.10 and follows with the stated hypothesis.

$$\chi^2 = \sum_{i=1}^{k}\sum_{j=1}^{k}\frac{(n_{ij} - e_{ij})^2}{e_{ij}} \qquad 3.10$$

The equation 3.10 is a χ^2 distribution with the $(k-1)^2$ degrees of freedom.

For a two-, three- and four- state Markov Chain, $k = 2, 3 \textit{ and } 4$, respectively. If $\chi^2 > \chi^2_{\alpha,(k-1)^2}$, the sequence is said to confirm the Markov property, else, the sequence would not be treated as a Markovian chain (Yue-zhi et al., 2003). Consequently, the hypothesis test for establishing Markovian property is given as follows:

H_0: Successive transitions are independent (Current and next states are independent).

H_1: Successive transitions are not independent (Current and next state are dependent).

3.12 Model Specification

Following the assumptions of the MCMs, three main features are desired and they include the following:

1. To identify possible states
2. To identify possible transitions
3. To identify transition Probabilities.

3.13 Construction of Markov Chain Models for Probability Forecasting

The following assumptions were made to enable the application of MCMs in predicting the probability of stock market returns;

1. The probability; with which market index returns move from one state i to another state j, by the same time interval; has nothing to do with the moment of state i.
2. The activities of the stock market, from which we derived the index, are impacted by random factors; such as, the state of local and global economic conditions, political environments, monetary and fiscal policy decision is stable and the various manipulated impacts of investors are negligible.
3. The movement of the market index returns, in a giving day, only depends on the state before the immediate closing day, and has little to do with the past; so the returns on the past are negligible.

3.14 Construction of a Markov Chain Model

If we have a collection of sequence data $\{ASIR_{(t)}\}$, then we can find the transition frequency n_{ij} in the sequence by counting the number of transitions from state i to state j in one step. We proceed ahead to construct the one step transition matrix as follows;

$$n_{ij} = \begin{pmatrix} n_{11} & , & , & n_{1m} \\ , & , & , & , \\ , & , & , & , \\ n_{m1} & , & , & n_{mm} \end{pmatrix} \tag{3.11}$$

and from n_{ij}, we proceed to get the estimate of p_{ij}, the transition probability as follows;

$$p_{ij} = \begin{pmatrix} p_{11} & , & , & p_{1m} \\ , & , & , & , \\ , & , & , & , \\ p_{m1} & , & , & p_{mm} \end{pmatrix} \tag{3.12}$$

where

$$p_{ij} = \begin{cases} \dfrac{n_{ij}}{\sum_{j=1}^{m} n_{ij}} & \text{if } \sum_{j=1}^{m} n_{ij} > 0 \\ 0 & \text{if } \sum_{j=1}^{m} n_{ij} = 0 \end{cases} \tag{3.13}$$

3.15 Possible States

The behaviour of SMRs is characterized into five different scenarios; first scenario is a 2-state, which was defined as low and high returns regime; second scenario is a 3-state regime, which was defined as low, medium and high returns regimes, based on Mean $\pm$ 1SD (standard deviation) classification; the third scenario is a 3-state regime, which was defined as low, medium and high returns regimes based on Quartiles(Q) classification; the fourth scenario is a 4-state regime, defined as strong-low, low, high and strong-high returns based on Mean $\pm$ 1SD classification; and the fifth scenario being a 4-state regime, defined as strong-low, low, high and strong-high returns based on Quartiles classification. These are applied to the three variables - All Share Index Returns (ASIR), Dangote Cement Plc Returns (DANGCEMR) and Guaranty Trust Bank Returns (GTBR).

3.16 Applying the threshold

In this research, the applications of the respective threshold are based on the mean, standard deviation and quartiles, to classify the respective states for the Scenarios 2 to 5.

3.17 Possible Scenarios

Scenario 1

This is a two-state Markov chains model, in which the two possible states - low and high returns, are identified.

$$S_j = \begin{cases} S_1, & \text{if} \quad \text{ASIR}_t < 0 \\ S_2, & \text{if} \quad ASIR_t \geq 0 \end{cases} \qquad 3.14$$

$$S_j = \begin{cases} S_1, & \text{if} \quad \text{DANGCEMR}_t < 0 \\ S_2, & \text{if} \quad \text{DANGCEMR}_t \geq 0 \end{cases} \qquad 3.15$$

$$S_j = \begin{cases} S_1, & \text{if} \quad \text{GTBR}_t < 0 \\ S_2, & \text{if} \quad \text{GTBR}_t \geq 0 \end{cases} \qquad 3.16$$

From equations 3.14 to 3.16, if $ASIR_t < 0$, $DANGCEMR_t < 0$ and $GTBR_t < 0$, S_j is classified as low returns state, while $ASIR_t \geq 0$, $DANGCEMR_t \geq 0$ and $GTBR_t \geq 0$ indicates S_j as high returns state.

Scenario 2

In this case, a three-state Markov chains model is considered, with the three states classified as $S = \{\text{Low Returns, Medium Returns, High Returns}\}$, such that the index classification is 1 for low returns, 2 for moderate returns and 3 for high returns. The states are thus defined as $S=\{1, 2, 3\}$. Therefore, the daily returns are transformed to states, according to equations 3.17, 3.18 and 3.19.

$$S_j = \begin{cases} S_1, & \text{if} \quad \text{ASIR}_t < \widehat{\mu}_{ASIR_t} - \widehat{\sigma}_{ASIR_t} \\ S_2, & \text{if} \quad \mu_{ASIR_t} - \widehat{\sigma}_{ASIR_t} \leq ASIR_t < \widehat{\mu}_{ASIR_t} + \widehat{\sigma}_{ASIR_t} \\ S_3, & \text{if} \quad \text{ASIR}_t \geq \widehat{\mu}_{ASIR_t} + \widehat{\sigma}_{ASIR_t} \end{cases} \qquad 3.17$$

$$S_j = \begin{cases} S_1, & \text{if} \quad \text{DANGCEMR}_t < \widehat{\mu}_{DANGCEMR_t} - \widehat{\sigma}_{DANGCEMR_t} \\ S_2, & \text{if} \quad \widehat{\mu}_{DANGCEMR_t} - \widehat{\sigma}_{DANGCEMR_t} \leq DANGCEMR_t < \widehat{\mu}_{DANGCEMR_t} + \widehat{\sigma}_{DANGCEMR_t} \\ S_3, & \text{if} \quad \text{DANGCEMR}_t \geq \widehat{\mu}_{DANGCEMR_t} + \widehat{\sigma}_{DANGCEMR_t} \end{cases} \qquad 3.18$$

$$S_j = \begin{cases} S_1, & \text{if} \quad \text{GTBR}_t < \widehat{\mu}_{GTBR_t} - \widehat{\sigma}_{GTBR_t} \\ S_2, & \text{if} \quad \widehat{\mu}_{GTBR_t} - \widehat{\sigma}_{GTBR_t} \leq GTBR_t < \widehat{\mu}_{GTBR_t} + \widehat{\sigma}_{GTBR_t} \\ S_3, & \text{if} \quad \text{GTBR}_t \geq \widehat{\mu}_{GTBR_t} + \widehat{\sigma}_{GTBR_t} \end{cases} \qquad 3.19$$

From equations 3.17 to 3.19, if $ASIR_t < \widehat{\mu}_{ASIR_t} - \widehat{\sigma}_{ASIR_t}$, $DANGCEMR_t < \widehat{\mu}_{DANGCEMR_t} - \widehat{\sigma}_{DANGCEMR_t}$ and $GTBR_t < \widehat{\mu}_{GTBR_t} - \widehat{\sigma}_{GTBR_t}$, the state is classified as low return state. The state is classified as medium returns state if $\widehat{\mu}_{DANGCEMR_t} - \widehat{\sigma}_{DANGCEMR_t} \leq DANGCEMR_t < \widehat{\mu}_{DANGCEMR_t} + \widehat{\sigma}_{DANGCEMR_t}$, $\widehat{\mu}_{ASIR_t} - \widehat{\sigma}_{ASIR_t} \leq ASIR_t < \widehat{\mu}_{ASIR_t} + \widehat{\sigma}_{ASIR_t}$ and $\widehat{\mu}_{GTBR_t} - \widehat{\sigma}_{GTBR_t} \leq GTBR_t < \widehat{\mu}_{GTBR_t} + \widehat{\sigma}_{GTBR_t}$. The state is classified as high returns state if $ASIR_t \geq \widehat{\mu}_{ASIR_t} + \widehat{\sigma}_{ASIR_t}$, $DANGCEMR_t \geq \widehat{\mu}_{DANGCEMR_t} + \widehat{\sigma}_{DANGCEMR_t}$ and $GTBR_t \geq \widehat{\mu}_{GTBR_t} + \widehat{\sigma}_{GTBR_t}$.

Scenario 3

This scenario is also characterized as a three-state Markov chains model, such that the three states are classified as follows: $S = \{\text{Low Returns, Medium Returns, High Returns}\}$ and having indexes 1, 2 and 3 corresponding to low, moderate and high returns, respectively, such that the states are now defined as S={ 1, 2, 3}. The daily returns are transformed to states, using equations 3.20, 3.21 and 3.22;

$$S_j = \begin{cases} S_1, & \text{if} \quad \text{ASIR}_t < \widehat{Q}_1 \\ S_2, & \text{if} \quad \widehat{Q}_1 \leq ASIR_t < \widehat{Q}_3 \\ S_3, & \text{if} \quad \text{ASIR}_t \geq \widehat{Q}_3 \end{cases} \qquad 3.20$$

$$S_j = \begin{cases} S_1, & \text{if} \quad \text{DANGCEMR}_t < \hat{Q}_1 \\ S_2, & \text{if} \quad \hat{\text{Q}}_1 \leq DANGCEMR_t < \hat{Q}_3 \\ S_3, & \text{if} \quad \text{DANGCEMR}_t \geq \hat{Q}_3 \end{cases} \qquad 3.21$$

$$S_j = \begin{cases} S_1, & \text{if} \quad \text{GTBR}_t < \hat{Q}_1 \\ S_2, & \text{if} \quad \hat{\text{Q}}_1 \leq GTBR_t < \hat{Q}_3 \\ S_3, & \text{if} \quad \text{GTBR}_t \geq \hat{Q}_3 \end{cases} \qquad 3.22$$

From equations 3.20 to 3.22, if $ASIR_t < \hat{Q}_1$, $DANGCEMR_t < \hat{Q}_1$ and $GTBR_t < \hat{Q}_1$, the state is classified as low returns state; when $\hat{Q}_1 \leq ASIR_t < \hat{Q}_3$, $\hat{Q}_1 \leq DANGCEMR_t < \hat{Q}_3$ and $\hat{Q}_1 \leq GTBR_t < \hat{Q}_3$, it is classified as medium returns state; and finally, when $ASIR_t \geq \hat{Q}_3$, $DANGCEMR_t \geq \hat{Q}_3$ and $GTBR_t \geq \hat{Q}_3$, it is classified as high returns state.

Scenario 4

This scenario is a four-state Markov chains model, with the classification of the four states as $S = \{Strong\ low\ return,\ low\ return,\ High\ return,\ Strong\ high\ return\}$ and the index classified as A, B, C and D corresponding to "strong-low returns", "low returns", "high returns" and "strong-high returns", respectively, such that the states are now defined as $S = \{\text{A, B, C,D}\}$. Consequently, the daily returns can be transformed to states, using equations 3.23, 3.24 and 3.25 given below;

$$S_j = \begin{cases} S_A, & \text{if ASIR}_t < \hat{\mu}_{ASIR_t} - \hat{\sigma}_{ASIR_t} \\ S_B, & \text{if } \hat{\mu}_{ASIR_t} - \hat{\sigma}_{ASIR_t} \leq ASIR_t < \hat{\mu}_{ASIR_t} \\ S_C, & \text{if } \hat{\mu}_{ASIR_t} \leq ASIR_t < \hat{\mu}_{ASIR_t} + \hat{\sigma}_{ASIR_t} \\ S_D, & \text{if ASIR}_t \geq \hat{\mu}_{ASIR_t} + \hat{\sigma}_{ASIR_t} \end{cases} \qquad 3.23$$

$$S_j = \begin{cases} S_A, & \text{if DANGCEMR}_t < \hat{\mu}_{DANGCEMR_t} - \hat{\sigma}_{DANGCEMR_t} \\ S_B, & \text{if } \hat{\mu}_{DANGCEMR_t} - \hat{\sigma}_{DANGCEMR_t} \leq DANGCEMR_t < \hat{\mu}_{DANGCEMR_t} \\ S_C, & \text{if } \hat{\mu}_{DANGCEMR_t} \leq DANGCEMR_t < \hat{\mu}_{DANGCEMRR_t} + \hat{\sigma}_{DANGCEMR_t} \\ S_D, & \text{if DANGCEMR}_t \geq \hat{\mu}_{DANGCEMR_t} + \hat{\sigma}_{DANGCEMR_t} \end{cases} \qquad 3.24$$

$$S_j = \begin{cases} S_A, & \text{if } \mathrm{GTBR}_t < \widehat{\mu}_{GTBR_t} - \widehat{\sigma}_{GTBR_t} \\ S_B, & \text{if } \widehat{\mu}_{GTBR_t} - \widehat{\sigma}_{GTBR_t} \le GTBR_t < \widehat{\mu}_{GTBR_t} \\ S_C, & \text{if } \widehat{\mu}_{GTBR_t} \le GTBR_t < \widehat{\mu}_{GTBR_t} + \widehat{\sigma}_{GTBR_t} \\ S_D, & \text{if } \mathrm{GTBR}_t \ge \widehat{\mu}_{GTBR_t} + \widehat{\sigma}_{GTBR_t} \end{cases} \qquad 3.25$$

From equations 3.23 to 3.25, if $DANGCEMR_t < \widehat{\mu}_{DANGCEMR_t} - \widehat{\sigma}_{DANGCEMR_t}$, $ASIR_t < \widehat{\mu}_{ASIR_t} - \widehat{\sigma}_{ASIR_t}$ and $GTBR_t < \widehat{\mu}_{GTBR_t} - \widehat{\sigma}_{GTBR_t}$, it is classified as strong-low returns state; if on the other hand, $\widehat{\mu}_{ASIR_t} - \widehat{\sigma}_{ASIR_t} \le ASIR_t < \widehat{\mu}_{ASIR_t}$, $\widehat{\mu}_{DANGCEMR_t} - \widehat{\sigma}_{DANGCEMR_t} \le DANGCEMR_t \widehat{\mu}_{DANGCEMR_t}$ and $\widehat{\mu}_{GTBR_t} - \widehat{\sigma}_{GTBR_t} \le GTBR_t < \widehat{\mu}_{GTBR_t}$, it is classified as low returns state; while for $\widehat{\mu}_{ASIR_t} \le ASIR_t < \widehat{\mu}_{ASIR_t} + \widehat{\sigma}_{ASIR_t}$, $\widehat{\mu}_{DANGCEMR_t} \le DANGCEMR_t < \widehat{\mu}_{DANGCEMR_t} + \widehat{\sigma}_{DANGCEMR_t}$ and $\widehat{\mu}_{GTBR_t} \le GTBR_t < \widehat{\mu}_{GTBR_t} + \widehat{\sigma}_{GTBR_t}$, it is classified as high returns state; and for $ASIR_t \ge \widehat{\mu}_{ASIR_t} + \widehat{\sigma}_{ASIR_t}$, $DANGCEMR_t \ge \widehat{\mu}_{DANGCEMR_t} + \widehat{\sigma}_{DANGCEMR_t}$ and $GTBR_t \ge \widehat{\mu}_{GTBR_t} + \widehat{\sigma}_{GTBR_t}$, it is classified as high returns state.

Scenario 5

For the fifth scenario, there are four states in the Markov chains model, with the states classified as $S = \{Strong\ low\ return,\ low\ return,\ High\ return,\ Strong\ High\ return\}$ and the indexes A, B, C and D, corresponding to “Strong-low returns”, “Low returns”, “High returns” and “Strong-high returns”, respectively. The states are now defined as $S=\{\text{A, B, C,D}\}$ and the daily returns are subsequently transformed into states using equations 3.26, 3.27 and 3.28, as given below.

$$S_j = \begin{cases} S_A, & \text{if } \mathrm{ASIR}_t < \widehat{Q}_1 \\ S_B, & \text{if } \widehat{Q}_1 \le ASIR_t < \widehat{Q}_2 \\ S_C, & \text{if } \widehat{Q}_2 \le ASIR_t < \widehat{Q}_3 \\ S_D, & \text{if } \mathrm{ASIR}_t \ge \widehat{Q}_3 \end{cases} \qquad 3.26$$

$$S_j = \begin{cases} S_A, & \text{if DANGCEMR}_t < \widehat{Q}_1 \\ S_B, & \text{if } \widehat{Q}_1 \le DANGCEMR_t < \widehat{Q}_2 \\ S_C, & \text{if } \widehat{Q}_2 \le DANGCEMR_t < \widehat{Q}_3 \\ S_D, & \text{if DANCEMR}_t \ge \widehat{Q}_3 \end{cases} \quad 3.27$$

$$S_j = \begin{cases} S_A, & \text{if GTBR}_t < \widehat{Q}_1 \\ S_B, & \text{if } \widehat{Q}_1 \le GTBR_t < \widehat{Q}_2 \\ S_C, & \text{if } \widehat{Q}_2 \le GTBR_t < \widehat{Q}_3 \\ S_D, & \text{if GTBR}_t \ge \widehat{Q}_3 \end{cases} \quad 3.28$$

From equations 3.26 to 3.28, if $ASIR_t < \widehat{Q}_1$, $DANGCEMR_t < \widehat{Q}_1$ and $GTBR_t < \widehat{Q}_1$, it is classified as strong-low returns state; if $\widehat{Q}_1 \le ASIR_t < \widehat{Q}_2$, $\widehat{Q}_1 \le DANGCEMR_t < \widehat{Q}_2$ and $\widehat{Q}_1 \le GTBR_t < \widehat{Q}_2$, it is classified as low returns state; if $\widehat{Q}_2 \le ASIR_t < \widehat{Q}_3$, $\widehat{Q}_2 \le DANGCEMR_t < \widehat{Q}_3$ and $\widehat{Q}_2 \le GTBR_t < \widehat{Q}_3$, it is classified as high returns state; while when $ASIR_t \ge \widehat{Q}_3$, $DANGCEMR_t \ge \widehat{Q}_3$ and $GTBR_t \ge \widehat{Q}_3$, it is classified as strong-high returns state.

3.18 Possible Transitions

For Scenario 1 – the case of two states, there are four (4) possible transitions, which are;

Low return → Low return

Low return → High return

High return → Low return

High return → High return

For Scenarios 2 and 3**,** both having 3-states model, there are nine (9) possible transitions. These include the following

Low return → Low return

Lower return → Medium return

Low return → High return

Medium return → Low return

Medium return → Medium return

Medium return → High return

High return → Low return

High return → Medium return

High return → High return

For Scenarios 4 and 5, both characterized as four-state models, there are sixteen (16) possible transitions. They include;

Strong low return → Strong low return

Strong low return → Low return

Strong low return → High return

Strong low return → Strong high return

Low return → Strong low return

Low return → Low return

Low return → High return

Low return → Strong high return

High return → Strong low return

High return → Low return

High return → High return

High return → Strong high return

Strong high return → Strong low return

Strong high return → Low return

Strong high return → High return

Strong high return → Strong high return

3.19 The Transition Probabilities

The use of transition probabilities provides the necessary information for the transition behaviour of the Markov chain model. The elements of transition probability matrix show the probability of transitions from a particular state to another state. This means that the transition probability refers to the probability of occurrence of a typical state from one of the existing states. The transition probabilities enhance the formation of an idea about the likelihood of occurrence of a future state of an event, and provide opportunity to make a guided decision accordingly (Madhav,2017).

3.20 Estimating the Transition Probabilities

To estimate the probability of transition, the maximum likelihood method was applied, and is defined in equation 3.29 as

$$P_{ij} = P(ASIR_{t+1} = j / ASIR_t = i) \qquad 3.29$$

Let $asir_i \leq asir_1, asir_2,,, \text{asir}_n$ be a realization of the random variable $ASIR_1^n$, with the probability of this realization defined by equation 3.30

$$P(ASIR_1^n = asir_1^n = P(ASIR_1 = asir_1)\prod_{t=2}^{n} P(ASIR_t = asir_t / ASIR_1^{t-1} = asir_1^{t-1}) \qquad 3.30$$

Also, the equation 3.30 regarding to the probabilities P_{ij} of transition can be written such that the likelihood for a transition probability matrix is given by;

$$L(p) = P(ASIR_1 = asir_1)\prod_{t=2}^{n} P_{(asir_{t-1})(asir_t)} \qquad 3.31$$

$$L(p) = P(ASIR_1 = asir_1)\prod_{i=1}^{k}\prod_{j=1}^{k} P_{ij}^{n_{ij}} \qquad 3.32$$

where n_{ij} is the number of times i transits to j in $asir_1^n$, such that equation 3.32 is the likelihood in terms of n_{ij}. Maximizing the likelihood function with respect to P_{ij} and taking the logarithm then results in equation 3.33

$$l(p) = \log L(p) = \log P(ASIR_1 = asir_1) + \sum_{i,j} n_{ij} \log P_{ij}$$

$$\Rightarrow \frac{\partial l}{\partial P_{ij}} = \frac{n_{ij}}{P_{ij}} \quad 3.33$$

Equating the derivative with respect to P_{ij} and equating zero yields

$$\frac{n_{ij}}{P_{ij}} = 0$$

If the assumption $\sum_j P_{ij} = 1$ is taken into cognizance and one of the transition probabilities is used to solve for the others; the probability of approaching 1 for each i will be defined by $P_{i1} = 1 - \sum_{j=2}^{n} P_{ij}$, then, taking the derivatives of the likelihood will result in $\frac{\partial}{\partial P_{ij}}$

$$\frac{\partial l}{\partial P_{ij}} = \frac{n_{ij}}{P_{ij}} - \frac{n_{i1}}{P_{i1}}$$

By setting this equal to zero for the MLE $\hat{P}$,

$$\frac{n_{ij}}{\hat{P}_{ij}} = \frac{n_{i1}}{\hat{P}_{i1}}$$

$$\frac{n_{ij}}{n_{i1}} = \frac{\hat{P}_{ij}}{\hat{P}_{i1}} \quad 3.34$$

This holds $\forall j \neq 1$, and $\hat{P}_{ij} \propto n_{ij}$ with

$$\hat{P}_{ij} = \frac{n_{ij}}{\sum_j n_{ij}} \quad 3.35$$

This gives the MLE of the transition probability (P_{ij})

Scenario1

The computations of the probability transitions are as follows;

$$n_{ij} = \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix} \quad 3.36$$

Then, also have the following probabilities

$$p_{11} = \frac{n_{11}}{n_{11} + n_{12}}$$

$$p_{12} = \frac{n_{12}}{n_{11} + n_{12}}$$

$$p_{21} = \frac{n_{21}}{n_{21} + n_{22}}$$

$$p_{22} = \frac{n_{22}}{n_{21} + n_{22}} \quad 3.37$$

With its probability transition matrix given as

$$p_{ij} = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} \quad 3.38$$

Scenario2

The transition frequency matrix is given as;

$$n_{ij} = \begin{pmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{pmatrix} \quad 3.39$$

The computations of the probability transitions are as follows;

$$p_{11} = \frac{n_{11}}{n_{11} + n_{12} + n_{13}}$$

$$p_{12} = \frac{n_{12}}{n_{11} + n_{12} + n_{13}}$$

$$p_{13} = \frac{n_{13}}{n_{11} + n_{12} + n_{13}} \quad 3.40$$

$$\vdots$$

$$p_{13} = \frac{n_{33}}{n_{31} + n_{32} + n_{33}}$$

With its probability transition matrix given as

$$p_{ij} = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} \qquad 3.41$$

Scenario3

The transition frequency matrix is given;

$$n_{ij} = \begin{pmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{pmatrix} \qquad 3.42$$

The computations of the probability transitions are as follows;

$$\begin{aligned} p_{11} &= \frac{n_{11}}{n_{11}+n_{12}+n_{13}} \\ p_{12} &= \frac{n_{12}}{n_{11}+n_{12}+n_{13}} \\ p_{13} &= \frac{n_{13}}{n_{11}+n_{12}+n_{13}} \\ &\vdots \\ p_{13} &= \frac{n_{33}}{n_{31}+n_{32}+n_{33}} \end{aligned} \qquad 3.43$$

With its probability transition matrix given as

$$p_{ij} = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} \qquad 3.44$$

Scenario4

The transition frequency matrix is given as;

$$n_{ij} = \begin{pmatrix} n_{AA} & n_{AB} & n_{AC} & n_{AD} \\ n_{BA} & n_{BB} & n_{BC} & n_{BD} \\ n_{CA} & n_{CB} & n_{CC} & n_{CD} \\ n_{DA} & n_{DB} & n_{DC} & n_{DD} \end{pmatrix} \qquad 3.45$$

The computations of the probability transitions are

$$\begin{aligned} p_{AA} &= \frac{n_{AA}}{n_{AA}+n_{AB}+n_{AC}+n_{AD}} \\ p_{AB} &= \frac{n_{AB}}{n_{AA}+n_{AB}+n_{AC}+n_{AD}} \\ p_{AC} &= \frac{n_{AC}}{n_{AA}+n_{AB}+n_{AC}+n_{AD}} \\ p_{AD} &= \frac{n_{AD}}{n_{AA}+n_{AB}+n_{AC}+n_{AD}} \\ p_{BA} &= \frac{n_{BA}}{n_{BA}+n_{BB}+n_{BC}+n_{BD}} \\ &\vdots \\ p_{DD} &= \frac{n_{DD}}{n_{DA}+n_{DB}+n_{DC}+n_{DD}} \end{aligned} \qquad 3.46$$

With its probability transition matrix given as

$$P_{ij} = \begin{pmatrix} P_{AA} & P_{AB} & P_{AC} & P_{AD} \\ P_{BA} & P_{BB} & P_{BC} & P_{BD} \\ P_{CA} & P_{CB} & P_{CC} & P_{CD} \\ P_{DA} & P_{DB} & P_{DC} & P_{DD} \end{pmatrix} \qquad 3.47$$

Scenario5

The transition frequency matrix is given as follows

$$n_{ij} = \begin{pmatrix} n_{AA} & n_{AB} & n_{AC} & n_{AD} \\ n_{BA} & n_{BB} & n_{BC} & n_{BD} \\ n_{CA} & n_{CB} & n_{CC} & n_{CD} \\ n_{DA} & n_{DB} & n_{DC} & n_{DD} \end{pmatrix} \qquad 3.48$$

The computations of the probability transitions are

$$\begin{aligned} p_{AA} &= \frac{n_{AA}}{n_{AA}+n_{AB}+n_{AC}+n_{AD}} \\ p_{AB} &= \frac{n_{AB}}{n_{AA}+n_{AB}+n_{AC}+n_{AD}} \\ p_{AC} &= \frac{n_{AC}}{n_{AA}+n_{AB}+n_{AC}+n_{AD}} \\ p_{AD} &= \frac{n_{AD}}{n_{AA}+n_{AB}+n_{AC}+n_{AD}} \\ p_{BA} &= \frac{n_{BA}}{n_{BA}+n_{BB}+n_{BC}+n_{BD}} \\ &\vdots \\ p_{DD} &= \frac{n_{DD}}{n_{DA}+n_{DB}+n_{DC}+n_{DD}} \end{aligned} \qquad 3.49$$

With its probability transition matrix given as

$$P_{ij} = \begin{pmatrix} P_{AA} & P_{AB} & P_{AC} & P_{AD} \\ P_{BA} & P_{BB} & P_{BC} & P_{BD} \\ P_{CA} & P_{CB} & P_{CC} & P_{CD} \\ P_{DA} & P_{DB} & P_{DC} & P_{DD} \end{pmatrix} \qquad 3.50$$

3.21 Transition Probability and Matrix of Probability Transition

The likelihood to move from one state to the next state, or stay in similar particular state for a single time frame is referred to as the probability of transition.

$$P_{ASIR_n ASIR_{n+1}} = P\{ASIR(t_{n+1}) = asir_{n+1} \mid ASIR(t_n) = asir_n\}$$ 3.51

Equation 3.51 is called the first order Markov chain probability of transition. This stands for the conditional likelihood of the system being in state $asir_{n+1}$ at time t_{n+1}, given that it had been in state $asir_n$ at time t_n.

The probabilities of the transitions can be organized in an $m \times m$ matrix. This matrix is called the probability matrix of one-step transition, as shown in equation 3.46 below;

$$P_{ij} = \begin{bmatrix} P_{11} & P_{12} & . & . & P_{1m} \\ P_{12} & P_{22} & . & . & P_{2m} \\ . & . & . & . & . \\ . & . & . & . & . \\ P_{m1} & P_{m2} & . & . & P_{mm} \end{bmatrix}$$ 3.52

where m represents the total of states. The matrix P is a square matrix that is non-negative for each element, with the elements in each row summing up to unity, that is, $\sum_{j=1}^{m} P_{ij} = 1,\ i = 1\ to\ m$ and $0 \le P_{ij} \le 1$.

The initial estimate of P_{ij} could be calculated as

$$P_{ij} = \frac{n_{ij}}{n_i}, \qquad i, j = 1, 2, \ldots, m$$

where n_{ij} is the sample size of the data that refers to the frequency of items or units transitioned from state i to state j. n_i is the sample of frequency data in state i. A matrix P, with non-negative components, such that the component in either row or column sum up to unity referred to as a stochastic or transition matrix. Here, the total of number of rows is equal to the total of number of columns, and this gives a complete Markov process description.

3.22 n-Step State Probabilities of Transition.

Let $P_{ij}(n)$ denote the conditional probability that at a particular time, the system will be in state j after n transitions, given that it is currently in state i. According to Ross (1989), this is defined as follows;

$$P_{ij} = P(ASIR_{m+n} = j / ASIR_n = i)$$

$$P_{ij}(0) = \begin{cases} 1, & \text{i=j} \\ 0, & \text{i} \neq \text{j} \end{cases}$$

$$P_{ij}(1) = P_{ij}$$

$P_{ij}(n)$ is a system that is in state i at time $t = n$. The likelihood that the system moves to state j at time $t = m + n$ (sometimes, these time periods are recognized as the total number of steps).

The probabilities $P_{ij}^{(n)}$ of n-step transition may be indicated as follows;

$$P^n = \begin{bmatrix} P_{11}^{(n)} & P_{12}^{(n)} & . & . & P_{1m}^{(n)} \\ P_{21}^{(n)} & P_{22}^{(n)} & . & . & P_{2m}^{(n)} \\ . & . & . & . & . \\ . & . & . & . & . \\ P_{m1}^{(n)} & P_{m2}^{(n)} & . & . & P_{mm}^{(n)} \end{bmatrix} \quad 3.53$$

From equation 3.53, P_{21}^{n} is the likelihood that a system that is present at state 2 moves to state 1, after steps “n”.

Theorem3.2

n-Step transition probability matrix $P^{(n)} = P^n$ where P^n is the n^{th} power of the matrix, P

Proof:

Since $P^0 = 1$ and $P^1 = P$, the theorem is true for $n = 0, 1$. The case of $n \geq 2$ will be as follows;

$$
\begin{aligned}
P_{ij}^{n} &= P(ASIR_n = j / ASIR_0 = i) \\
&= \sum_{k=1}^{N} P(ASIR_n = j / ASIR_{n-1} = k, ASIR_0 = i)P(ASIR_{n-1} = k / ASIR_0 = i) \\
&= \sum_{k=1}^{N} P_{ik}^{(\text{n}-1)} P(ASIR_n = j / ASIR_{n-1} = k, ASIR_0 = i) \\
&= \sum_{k=1}^{N} P_{ik}^{(\text{n}-1)} P(ASIR_n = j / ASIR_{n-1} = k) \qquad \left(due\ to\ Markov\ property\right) \qquad 3.54 \\
&= \sum_{k=1}^{N} P_{ik}^{(\text{n}-1)} P(ASIR_1 = j / ASIR_0 = k) \qquad \left(due\ to\ time\ homogeneity\right) \\
&= \sum_{k=1}^{N} P_{ik}^{(\text{n}-1)} P_{kj}
\end{aligned}
$$

The last sum followed the matrix multiplication operation, which is valid for $1 \le i, j \le N$. For a two-step transition $P_{ij}(2)$, which is defined as follows;

$$P_{ij}(2) = P(ASIR_{m+2} = j / ASIR_n = i)$$

Assume that m=0, then we have

$$
\begin{aligned}
P_{ij}(2) &= P(ASIR_2 = j / ASIR_0 = i) \\
&= \sum_{k} P(ASIR_2 = j, \text{ASIR}_1 = k / ASIR_0 = i) \\
&= \sum_{k} P(ASIR_2 = j / ASIR_1 = k, ASIR_0 = i)P(ASIR_1 = k / ASIR_0 = i) \\
&= \sum_{k} P(ASIR_2 = j / ASIR_1 = k)P(ASIR_1 = k / ASIR_0 = i) \qquad 3.55 \\
&= \sum_{k} P_{kj} P_{ik} \\
&= \sum_{k} P_{ik} P_{kj}
\end{aligned}
$$

3.23 Chapman-Kolmogorov Equations

The Chapma-Kolmogrov equations provide a method for computing the n-step transition probabilities. The one-step transition probability p_{ij}, defined in equation 3.54, on the basis that from the probabilities of $n-step$, such n may be obtained from the probabilities of $1-step$. Therefore, the $n-step$ transition probabilities p_{ij}^{n} can be defined as the probability that a process in state i will be in state j, after n additional transitions. According to Ross (2014), it was defined as follows;

$$P_{ij}^{n} = P\left(ASIR_{n+k} = j \middle| ASIR_{k} = i\right) \qquad n \geq 0,\ i, j \geq 0 \tag{3.56}$$

However, note that $P_{ij}^{1} = P_{ij}$, while the $n-step$ transition probabilities are obtained as follows;

$$P_{ij}^{n+m} = \sum_{k=0}^{\infty} P_{ik}^{n} P_{kj}^{m} \qquad \forall n, m \geq 0,\ \forall i, j \tag{3.57}$$

One should however understand that $P_{ik}^{n} P_{kj}^{m}$ represents for the probability that starting from state i, the process will move to state j in $n+m$ transitions, on a path that takes it into state k at the n^{th} transition. Summing it together with the intermediate states k will yield the probability that the process will be in state j after $n+m$ transitions. Mathematically, it follows that;

$$\begin{aligned} P_{ij}^{n+m} &= P\left(ASIR_{n+m} = j \middle| ASIR_{0} = i\right) \\ &= \sum_{k=0}^{\infty} P\left(ASIR_{n+m} = j, ASIR_{n} = k \middle| ASIR_{0} = i\right) \\ &= \sum_{k=0}^{\infty} P\left(ASIR_{n+m} = j \middle| ASIR_{n} = k,\ ASIR_{0} = i\right) P\left(ASIR_{n} = k \middle| ASIR_{0} = i\right) \\ &= \sum_{k=0}^{\infty} P_{kj}^{m} P_{ik}^{n} \end{aligned} \tag{3.58}$$

Also let $P^{(n)}$ represent the matrix of $n-step$ transition probabilities, P_{ij}^{n}; then equation 3.58 establishes that $P^{(n+m)} = P^{(n)} * P^{(m)}$, which means that;

$$P^{(2)} = P^{(1+1)} = P * P = P^{2}$$

$$P^{(3)} = P^{(2+1)} = P^{2} * P^{1} = P^{3}$$

In the same process, by induction, the following relationship exists;

$$P^{(n)} = P^{(n-1+1)} = P^{(n-1)} \times P = P^{(n)}$$ 3.59

One can obtained the $n-step$ transition matrix by multiplying the matrix P by itself n times (assuming that the transition probability is homogeneous).

3.24 Stochastic Matrix

A Stochastic matrix is a square matrix P, which satisfies the condition $P_{ij} \geq 0, \; \forall i, j$. For each row i, $\sum_j P_{ij} = 1$. This is also referred to as the Regular Transition Matrix.

3.25 State of Markov Chains

For a better knowledge of state of Markov chains, MC consists of several transient and recurrent classes. But in the course of this study, the accessibility of the states from each other is addressed; the possibility of moving from state i to state j. Here, it can be said that the state j is accessible from state, this leads to the following definitions

Definition 3.4

A state j is accessible from state i, written as $i \leftrightarrow j$, if $P_{ij}^{n} > 0$ for some n. Every state is assumed to be accessible from itself since $P_{ii}^{(0)} = 1$.

Definition 3.5

Two states i and j are said to communicate, when written as $i \leftrightarrow j$, and if they are accessible from each other. In other words, $i \leftrightarrow j$ also means $j \leftrightarrow i$.

Definition 3.6

A Markov chain is said to be irreducible if all states communicate with each other.

3.26 Transient and Recurrent State

If a chain that is in state i, one might need to ask that if the chain will ever return to state i. An affirmation answer implies that state i is a recurrent state, otherwise, it is a transient state. Further definition with the notion of hitting time (H_{ij}), is given as

$$H_{ij} = \min\{k > 0 : \mathrm{ASIR}_0 = \mathrm{i}, \mathrm{ASIR}_k = j\}$$

The hitting time represents the first time the chain enters state j, given that it started from state i. If $j = i$, then H_{ii} is the first time that the chain returns to state i, given that it is currently in state i. The random variable H_{ii} is therefore referred to as the recurrence time of state i.

As for discrete time, H_{ii} takes the value 1, 2,…. Furthermore, P_i^k is defined as the probability that the recurrence time of the state i is k; such that

$$P_i^k \equiv P[H_{ii} = k]$$

P_i is the probability that the event [ever return to i | current state is i], given by

$$P_i = \sum_{k=1}^{\infty} P_i^k$$

Given also that the event [ever return to i |current state is i] is similar to the event, $[H_{ii} < \infty]$,

$P_i = P[H_{ii} < \infty]$

Definition 3.7

A state i is referred to as recurrent if $P_i = 1$, while it is transient if $P_i < 1$.

In summary, recurrence means that a state is possibly visited again. A recurrent state is where an item keeps coming back to, and a transient state is one where an item never returns to. Every state is therefore, either recurrent or transient.

3.27 Absorbing States

A state i is called an absorbing state if it is impossible to leave this particular state. It further means that the state i is absorbing, if and only if, $p_{ii} = 1$ and $p_{ij} = 0$ for $i \neq j$. In a Markov chain, if every state can reach an absorbing state, then the Markov chain is an absorbing Markov chain. However, for a state that is not absorbing, the Markov chain is called transient. Having a discrete time absorbing Markov chain with a finite discrete-state-space and continuous-time absorbing Markov chains.

3.28 Steady State Probabilities

This is the probability that one approaches, after a large number of transition, is referred to as steady state probabilities. As n gets larger, the state probabilities at the $(\text{n+1})^{th}$ period get closer to those at the $(\text{n})^{th}$ period.

$$\lambda(n) = [\lambda_1(n), \lambda_2(n)]$$
$$[\lambda_1(n), \lambda_2(n)] = (\lambda_1(n), \lambda_2(n)) \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} \qquad 3.60$$

These probabilities are called steady state probabilities and are the long-term probability of being in a particular state, no matter the state one began.

3.29 Limiting Distribution

In a number of studies, a Markov chain exhibits a long-term limiting behaviour (Dobrow 2016). The chain is expected to settle down to an equilibrium distribution, which does not depend on the position of its initial state.

3.29.1 Expression of Limiting Distribution

Let $ASIR_0$, $ASIR_1, ASIR_2, \ldots$ be a transition matrix Markov chain P. A Markov chain limiting distribution is a probability distribution, λ, with the property that for all i and j, and mathematically expressed as

$$\lim_{n \to \infty} P_{ij} = \lambda_j$$

However, the above expression of the limiting distribution is the same as each of the following mathematical statements;

(1) For any initial distribution, for all j

$$\lim_{n \to \infty} P(ASIR_n = j) = \lambda_j \qquad 3.61$$

(2) For any initial distribution α, have

$$\lim_{n \to \infty} \alpha P^n = \lambda \qquad 3.62$$

(3) $\lim_{n \to \infty} P^n = \lambda$ 3.63

where λ is a stochastic matrix, equivalent to all rows.

3.30 Stationary Distribution

Let $ASIR_0$, $ASIR_1, ASIR_2, \ldots$ be a transition matrix P Markov chain. A stationary distribution is a distribution of probability λ that satisfies;

$$\lambda = \lambda P \qquad 3.64$$

that is $\lambda_j = \sum_i \lambda_i P_{ij}$ for all j.

On the assumption that a stationary distribution λ is the initial distribution, equation 3.61 implies that the distribution of $ASIR_0$ is similar to the distribution of $ASIR_1$.

$$\begin{aligned}\lambda P^n &= (\lambda P)P^{n-1} = \lambda P^{n-1} \\ &= (\lambda P)P^{n-2} = \lambda P^{n-2} \\ &= \ldots \lambda P = \lambda\end{aligned} \qquad 3.65$$

If $ASIR_0, ASIR_1, ASIR_2, \ldots$ is a stationary Markov chain, then for any $n > 0$, the sequence $ASIR_n, ASIR_{n+1}, ASIR_{n+2}, \ldots$ is also a stationary Markov chain; all of which have the same transition matrix as the initial chain. Stationary distribution can also be referred to as invariant, steady-state or equilibrium distribution.

3.30.1 Limiting Distributions are Stationary Distribution

Lemma

Assume that λ is the limiting distribution of a Markov chain, with transition matrix P, then λ is a stationary distribution.

Proof:

Assume λ is the limiting distribution, the aim is to establish that $\lambda P = \lambda$, for an initial distribution α

$$\begin{aligned}\lambda &= \lim_{n\to\infty} \alpha P^n \\ &= \lim_{n\to\infty} \alpha (P^{n-1}P) \\ &= \lim_{n\to\infty} (\alpha P^{n-1})P \\ &= \lambda P\end{aligned} \qquad 3.66$$

Having four (4) possible scenarios of the status of the limiting distributions as follows;

1. lim P^n exist, has identical rows and each row sum to one
2. lim P^n exist, does not have identical rows and each row sum to one.
3. lim P^n exist, but rows not sum to one
4. lim P^n does not exist

3.31 Mean Return Times of Markov Chains

The analysis and computation of mean return times of Markov chains is a direct by-product of the steady-state probabilities. The actual determination of the expected number of transitions before the system returns to a state j for the first time. This is known as the mean first return time or the mean recurrence time; computed in a $n-state$ Markov chains as;

$$\mu_j = \frac{1}{\lambda_j}, \quad j = 1, 2, \ldots, n \tag{3.67}$$

All things being equal, in the long-run mean recurrence time, the various states of the transitions will be computed as follows;

For $j = 2$

$$\mu_j = \frac{1}{\lambda_j} = [\frac{1}{\lambda_1}, \frac{1}{\lambda_2}] \tag{3.68}$$

For $j = 3$

$$\mu_j = \frac{1}{\lambda_j} = [\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}] \tag{3.69}$$

For $j = 4$

$$\mu_j = \frac{1}{\lambda_j} = [\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}, \frac{1}{\lambda_4}] \tag{3.70}$$

3.32 Occupancy Times

Let $\{ASIR_n, n \geq 0\}$ be a time-homogeneous DTMC on the state-space $S = \{1, 2, \ldots, N\}$, with transition probability matrix P and initial distribution $\alpha = [\alpha_1, \alpha_2, ...\alpha_N]$. Here, the focus is on occupancy times; the expected amount of time the DTMC spends in a given state, during a given interval of time, Kulkarni (2011). In other words, the expected time spent by the DTMC in various states. Let $V_j^{(n)}$ the number of visits to state j by the DTMC over $\{0,1,2,3,\ldots,n\}$. Noteworthy here, is the inclusion of the visit at zero, that is $V_j^0 = 1$ if $ASIR_0 = j$, and zero, otherwise.

3.31.1 Definition of Occupancy Times

$$M_{ij}^{(n)} = E(V_j^{(n)} / \text{ASIR}_0 = i) \qquad i, j \in s, n \geq 0 \tag{3.71}$$

The quantity $M_{ij}(n)$ is called the occupancy time up to n of state j, starting from state i.

$$M(\text{n}) = [M_{ij}(\text{n})] \tag{3.72}$$

$$M(n) = \begin{bmatrix} m_{11}(\text{n}) & m_{12}(\text{n}) & . & . & m_{1N}(\text{n}) \\ m_{21}(\text{n}) & m_{22}(\text{n}) & . & . & m_{2N}(\text{n}) \\ . & . & . & . & . \\ . & . & . & . & . \\ m_{N1}(\text{n}) & m_{N2}(\text{n}) & . & . & m_{NN}(\text{n}) \end{bmatrix} \tag{3.73}$$

However, let equation 3.72 be the occupancy times matrix; then, the theorem 3.2 gives the methodology of computing the occupancy times. Kulkarni (2011)

Theorem3.2 (Occupancy Times)

Let $\{ASIR_n, n \geq 0\}$ be a time-homogeneous DTMC on the state-space $S = \{1, 2, \ldots, N\}$ with transition probability matrix P. The occupancy times matrix is given as; $M(n) = \sum_{r=0}^{n} P^r$, $\text{n} \geq 0$ where $P^0 = 1$, the identity matrix.

Proof

Fix a $j \in s$, let $W_r = 1$ if $ASIR_r = j$ and zero, otherwise. Then,

$$V_j(n) = W_0 + W_1 + \ldots + W_n$$

$$V_j(n) = \sum_{r=0}^{n} W_r$$

Then, proceeds to get as follows;

$$\begin{aligned}
M_{ij}^{n} &= E\left(V_j^n \middle| ASIR_0 = i\right) \\
&= E\left(W_0 + W_1 + \ldots + W_n \middle| ASIR_0 = i\right) \\
&= E\left(\sum_{r=0}^{n} W_r \middle| ASIR_0 = i\right) \\
&= \sum_{r=0}^{n} E\left(W_r \middle| ASIR_0 = i\right) \\
&= \sum_{r=0}^{n} P\left(W_r = 1 \middle| ASIR_0 = i\right) \qquad 3.74 \\
&= \sum_{r=0}^{n} P\left(ASIR_r = j \middle| ASIR_0 = i\right) \\
&= \sum_{r=0}^{n} P_{ij}^{(r)} = \sum_{r=0}^{n} \left[P^{(r)}\right]_{ij} \\
&= \sum_{r=0}^{n} \left[P^{r}\right]_{ij}
\end{aligned}$$

For 2-state regime,

$$M(n) = \sum_{r=0}^{n} \begin{pmatrix} p_{12} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}^{n} \qquad 3.75$$

For 3-state regime,

$$M(n) = \sum_{r=0}^{n} \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix}^{n} \qquad 3.76$$

For 4-state regime,

$$M(n)=\sum_{r=0}^{n}\begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \\ p_{41} & p_{42} & p_{43} & p_{44} \end{pmatrix}^{n} \tag{3.77}$$

Equations 3.75, 3.76 and 3.77 give the value for occupancy times, that is, the expected amount of time the DTMC spends in a given state, during a given interval of time for 2-state, 3-state and 4-state regimes, respectively. For this research, $n=5$ (5 trading days) and $n=10$ (10 trading days) will be used to represent one and two weeks of trading, respectively.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Introduction

This chapter captures summary of statistics along with discussion of the results, which includes the classification of the states of the Markov models for the daily All-Share Index returns, and the returns of two other highly capitalized quoted stocks using 2-state, 3-state and 4-state models, with the transition probabilities for each of the states. In the study, the long-run distributions (stationary distributions) are found for the respective states, across the sample for the All Share Index, Dangote Cement and Guaranty Trust Bank Plc. Similarly, the recurrent time of the various states of returns are also obtained. The occupancy times were obtained along with the computations of the expected time spent by the discrete time Markov chain in various states.

This research study was based on the Markov chains application to the daily stock-market returns in Nigeria and two of the most capitalized stocks, as at June 30, 2018. Based on the research, the analysis showed the means of the daily returns of ASIR to be 0.0002, 0.0003 for DANGCEMR and 0.0004 for GTBR. With a total weekly data point of 122 weeks, the study reported 0.00588 weekly returns for ASIR (0.0010 for daily returns). This might be due to small sample size used. On the average, GTBR has higher returns than the market (ASIR) and DANGCEMR (Table 4.1). With the skewness value of -2.2733 for ASIR and -1.4002 for GTBR, both stocks appeared to be skewed to the left, with excess kurtosis of 1475.26 and 55.5504, respectively; indicating heavy tails, while the Nigerian stock market was Leptokurtic and Non-Gaussian (Okonta et al 2017). On the other hand, DANGCEMR with skewness value of 0.3189 is skewed to the right, with light heavy tail, given an excess kurtosis of 5.4401. The positive skewness of DANGCEMR indicates that the upper tail of the distribution is thicker than the lower, meaning that the returns here, rise more often than it declines. This case was different for ASIR and GTBR, with negative skewness; where the lower tail is thicker than upper tail, implying that the returns decline more often in this case. This aligns with the submission of Ngozi (2014).

Table 4.1: Summary of Statistics

	ASIR	DANGCEMR	GTBR
Mean	**0.0002**	**0.0003**	**0.0004**
Standard Deviation (SD)	**0.1166**	**0.0200**	**0.0306**
Q1 (Quartile 1)	**-0.0047**	**-0.0022**	**-0.0107**
Q2 (Quartile 2)	**0.0000**	**0.0000**	**0.0000**
Q3 (Quartile 3)	**0.0048**	**0.0028**	**0.0121**
Mean-SD	**-0.1164**	**-0.0197**	**-0.0302**
Mean+SD	**0.1167**	**0.0203**	**0.0310**
Skewness	**-2.2733**	**0.3189**	**-1.4002**
Kurtosis	**1475.2600**	**5.4401**	**55.5504**
n (Trading Days)	**3,066**	**1,867**	**3,066**

Source: Produced by the Author

In describing the daily index of Nigerian Stock Exchange All Share Index (ASI) for the period of this research, the graph of the daily index ASI are plotted against the trading days, as shown in Figure 4.1. From the graph, it is evident that the market recorded the highest value of index (66,371.20) on March 3, 2008. From a lower value of 24,085.71 in June 3, 2006, the market also recorded its lowest value of 22,800.70 in February 18, 2009. It is evident that the market trends in cycles; from low to high, low to low, high to low and high to high.

Also, from the daily index returns of Nigerian Stock Exchange All Share Index Return (ASIR) for the same period of this research, the graph of the daily index return ASIR is plotted against the trading days, as shown in Figure 4.2. From the graph, there was an evidence of transition from one state to another state. Between the years 2009 and 2012, persistent transition was observed.

The graph of the daily price of DANGCEM is plotted against the trading days, as shown in Figure 4.3. The cement company, which became a quoted company in October 2010, portrayed evidence that the market trends in cycles, from low to high, low to low, high to low and high to high. It reported the first high price of N210 on June 5, 2013, from the listing price of N135. However, it came below the price in September 2011. It moves to another, N240 on June 27, 2014, and reported another peak of N273 on January 17, 2018. From the study, DANGCEM had experienced many transitions, in both prices and returns, as captured in Figure 4.4.

The graph of the daily prices and returns of GTB are plotted against the trading days, as shown in Figures 4.5 and 4.6, respectively. It shows on the graphs that the market trends in cycles; from low to high, low to low, high to low and high to high. From the study, GTB had experience a sequence of transitions in both prices and returns, meaning that in the past, the investors of GTB had witness multiple periods of up and down.

Figure 4.1: NSE All Share Index 2006-2018 (y axis stand for index and x axis for years)

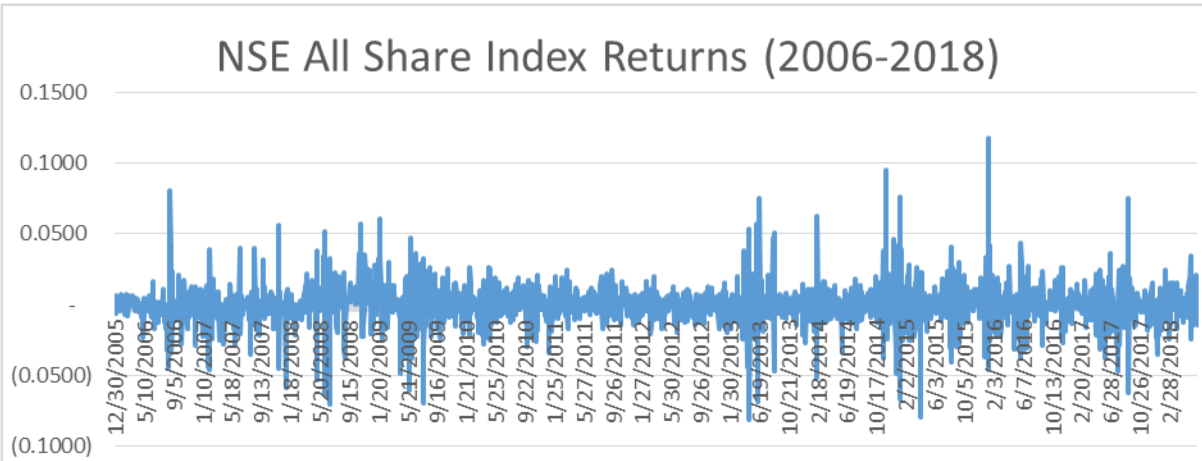

Figure 4.2: NSE All Share Index Returns 2006-2018 ;(y axis stand for returns and x axis for years)

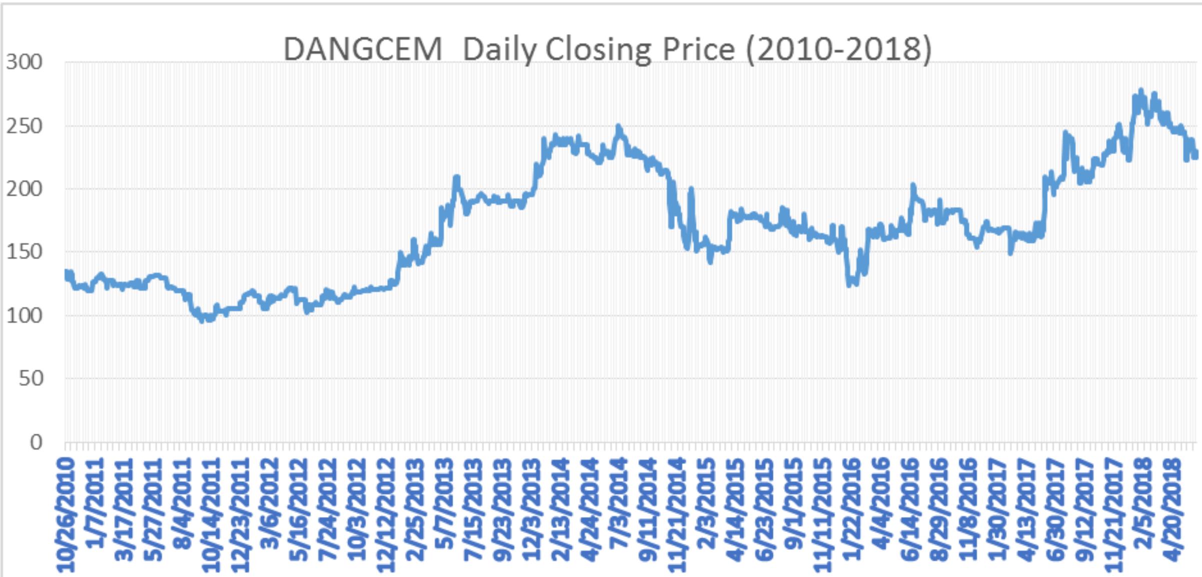

Figure 4.3: Dangcem Daily Closing Price 2010-2018. (y axis stands for prices and x axis for years)

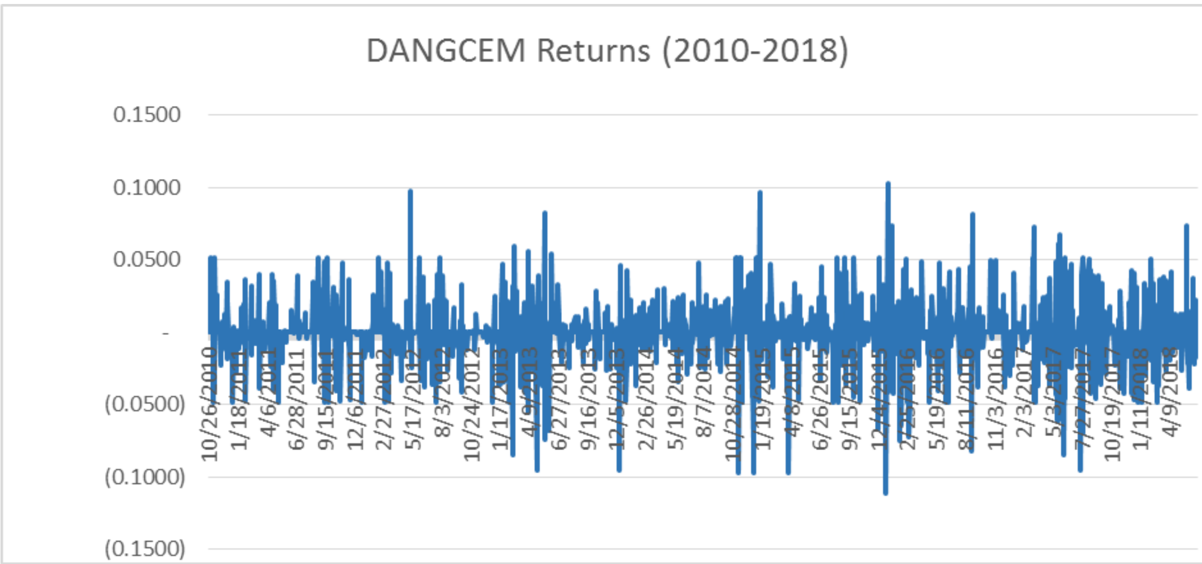

Figure 4.4: Dangcem Returns 2010-2018 . (y axis stands for returns and x axis for years)

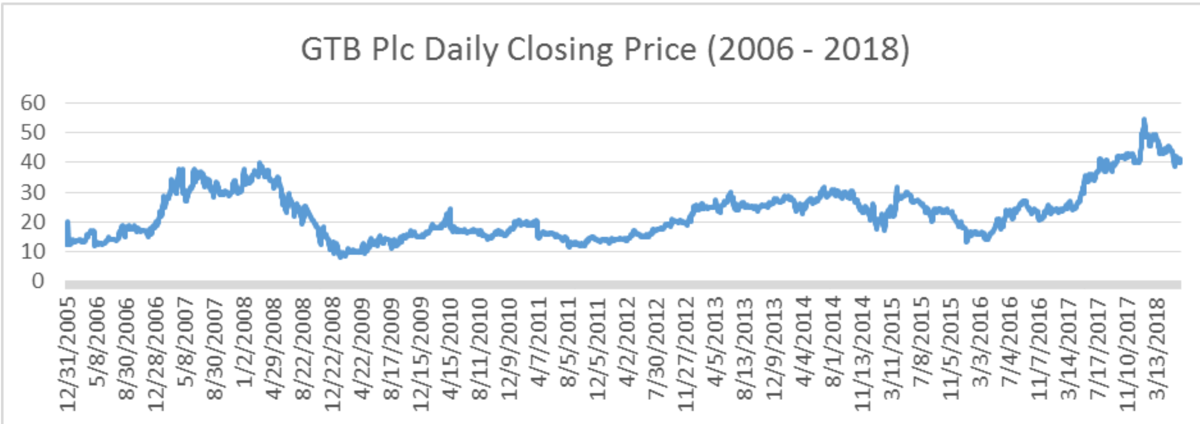

Figure 4.5: GTB Plc Closing Price 2006-2018   (y axis stand for prices and x axis for years)

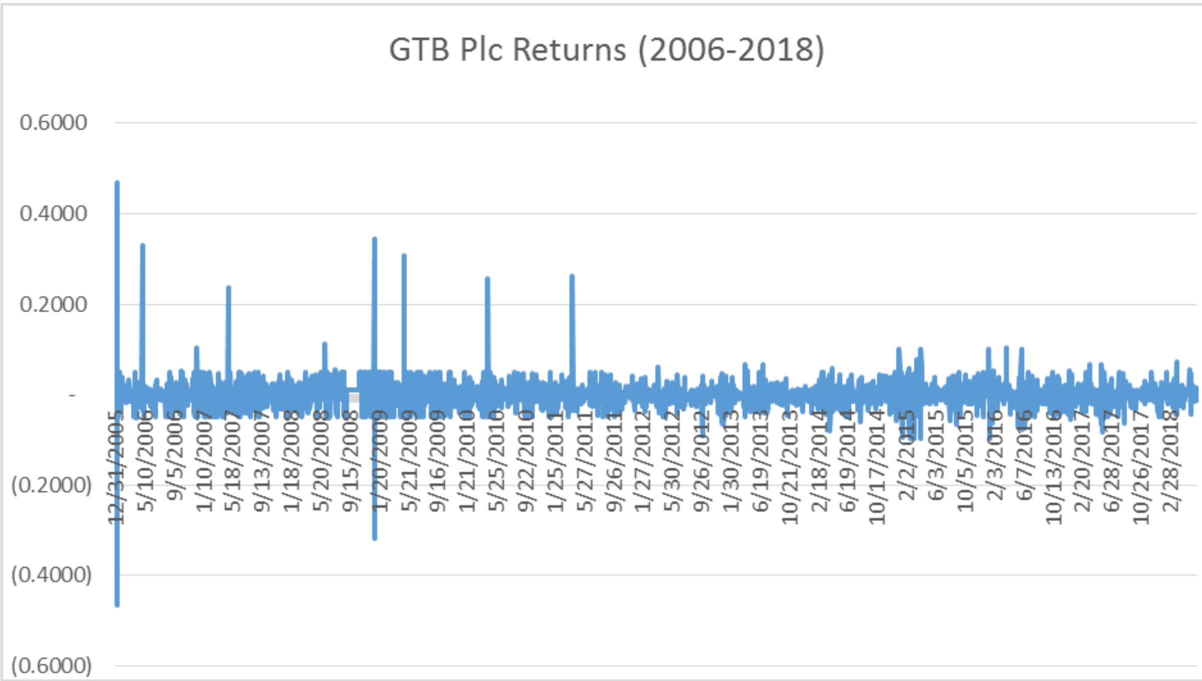

Figure 4.6: GTB Plc Returns 2006-2018. (y axis stand for returns and x axis for years)

4.2 Classification of states of returns.

Following the results obtained from Table 4.1, the classes of respective states of returns are constructed, in fulfillment of one of the assumptions of MCMs. For each of the scenario, the states of returns are as follows:

Scenario 1

$$S_j = \begin{cases} S_1, & \text{if} \quad \text{ASIR}_t < 0.0000 \\ S_2, & \text{if} \quad ASIR_t \geq 0.0000 \end{cases} \qquad 4.1$$

$$S_j = \begin{cases} S_1, & \text{if} \quad \text{DANGCEMR}_t < 0.0000 \\ S_2, & \text{if} \quad \text{DANGCEMR}_t \geq 0.0000 \end{cases} \qquad 4.2$$

$$S_j = \begin{cases} S_1, & \text{if} \quad \text{GTBR}_t < 0.0000 \\ S_2, & \text{if} \quad \text{GTBR}_t \geq 0.0000 \end{cases} \qquad 4.3$$

Here, taking the closing day as discrete to time units in equations 4.1 to 4.3, whenever the computed returns, $ASIR_t < 0$, $DANGCEMR_t < 0$, and $GTBR_t < 0$, it will be classified as low returns, respectively. Likewise when $ASIR_t \geq 0$, $DANGCEMR_t \geq 0$, and $GTBR_t \geq 0$, it will be classified as high returns.

Scenario 2

$$S_j = \begin{cases} S_1, & \text{if} \quad \text{ASIR}_t < -0.1164 \\ S_2, & \text{if} \quad \text{-0.1164} \leq ASIR_t < 0.1167 \\ S_3, & \text{if} \quad \text{ASIR}_t \geq 0.1167 \end{cases} \qquad 4.4$$

$$S_j = \begin{cases} S_1, & \text{if} \quad \text{DANGCEMR}_t < -0.0197 \\ S_2, & \text{if} \quad \text{-0.0197} \leq DANGCEMR_t < 0.0203 \\ S_3, & \text{if} \quad \text{DANGCEMR}_t \geq 0.0203 \end{cases} \qquad 4.5$$

$$S_j = \begin{cases} S_1, & \text{if} \quad \text{GTBR}_t < -0.0302 \\ S_2, & \text{if} \quad \text{-0.0302} \leq GTBR_t < 0.0310 \\ S_3, & \text{if} \quad \text{GTBR}_t \geq 0.0310 \end{cases} \qquad 4.6$$

For equations 4.4 to 4.6, representing scenario 2, where the threshold applied was Mean $\pm$ 1SD (standard deviation), the classifications are as follows; $ASIR_t < -0.1164$, $DANGCEMR_t < -0.0197$, and $GTBR_t < -0.0302$, were classified as low returns; $-0.1164 \leq ASIR_t < 0.1167$, $-0.0197 \leq DANGCEMR_t < 0.0203$, and $-0.0302 \leq GTBR_t < 0.0310$, the returns are classified as medium returns, respectively; while $ASIR_t \geq 0.1167$, $DANGCEMR_t \geq 0.0203$, and $GTBR_t \geq 0.0310$ are classified as high returns, for ASIR, DANGCEMR and GTBR, respectively.

Scenario 3

$$S_j = \begin{cases} S_1, & \text{if} \quad \text{ASIR}_t < -0.0047 \\ S_2, & \text{if} \quad \text{-0.0047} \leq ASIR_t < 0.0048 \\ S_3, & \text{if} \quad \text{ASIR}_t \geq 0.0048 \end{cases} \qquad 4.7$$

$$S_j = \begin{cases} S_1, & \text{if} \quad \text{DANGCEMR}_t < -0.0022 \\ S_2, & \text{if} \quad \text{-0.0022} \leq DANGCEMR_t < 0.0028 \\ S_3, & \text{if} \quad \text{DANGCEMR}_t \geq 0.0028 \end{cases} \qquad 4.8$$

$$S_j = \begin{cases} S_1, & \text{if} \quad \text{GTBR}_t < -0.0107 \\ S_2, & \text{if} \quad \text{-0.0107} \leq GTBR_t < 0.0121 \\ S_3, & \text{if} \quad GTBR_t \geq \ 0.0121 \end{cases} \qquad 4.9$$

Taking equations 4.7 to 4.9, which represent states in scenario 3, with the applied threshold that is based on Quartiles (Q); the classifications are as follows: $ASIR_t < -0.0047$, $DANGCEMR_t < -0.0022$, and $GTBR_t < -0.0107$ were classified as low returns, $-0.0047 \leq ASIR_t < 0.0048$, $-0.0022 \leq DANGCEMR_t < 0.0028$, and $-0.0107 \leq GTBR_t < 0.0121$ are classified as medium returns, while $ASIR_t \geq 0.0048$, $DANGCEMR_t \geq 0.0028$, and $GTBR_t \geq 0.0121$ are classified as high returns, respectively, for ASIR, DANGCEMR and GTBR.

Scenario 4

$$S_j = \begin{cases} S_A, & \text{if ASIR}_t < -0.1164 \\ S_B, & \text{if -0.1164} \le ASIR_t < 0.0002 \\ S_C, & \text{if } 0.0002 \le ASIR_t < 0.1167 \\ S_D, & \text{if ASIR}_t \ge 0.1167 \end{cases} \quad 4.10$$

$$S_j = \begin{cases} S_A, & \text{if DANGCEMR}_t < -0.0197 \\ S_B, & \text{if -0.0197} \le DANGCEMR_t < 0.0003 \\ S_C, & \text{if } 0.0003 \le DANGCEMR_t < 0.0203 \\ S_D, & \text{if DANGCEMR}_t \ge 0.0203 \end{cases} \quad 4.11$$

$$S_j = \begin{cases} S_A, & \text{if GTBR}_t < -0.0302 \\ S_B, & \text{if -0.0302} \le GTBR_t < 0.0004 \\ S_C, & \text{if } 0.0004 \le GTBR_t < 0.0310 \\ S_D, & \text{if GTBR}_t \ge 0.0310 \end{cases} \quad 4.12$$

Equations 4.10 to 4.12 represent the states classifications of the fourth scenario, where the threshold applied was Mean $\pm$ 1SD (Standard Deviation), and are as follows: $ASIR_t < -0.1164$, $DANGCEMR_t < -0.0197$, and $GTBR_t < -0.0302$ are classified as strong-low returns; $-0.1164 \le ASIR_t < 0.0002$, $-0.0197 \le DANGCEMR_t < 0.0003$, and $-0.0302 \le GTBR_t < 0.0004$ are classified as low returns; $0.0002 \le ASIR_t < 0.1167$, $0.0003 \le DANGCEMR_t < 0.0203$, and $0.0004 \le GTBR_t < 0.0310$ are classified as high returns; while $ASIR_t \ge 0.1167$, $DANGCEMR_t \ge 0.0203$ and $GTBR_t \ge 0.0310$ are classified as strong-high returns.

Scenario 5

$$S_j = \begin{cases} S_A, & \text{if ASIR}_t < -0.0047 \\ S_B, & \text{if -0.0047} \le ASIR_t < 0.0000 \\ S_C, & \text{if } 0.0000 \le ASIR_t < 0.0048 \\ S_D, & \text{if ASIR}_t \ge 0.0048 \end{cases} \quad 4.13$$

$$S_j = \begin{cases} S_A, & \text{if DANGCEMR}_t < -0.0022 \\ S_B, & \text{if -0.0022} \le DANGCEMR_t < 0.0000 \\ S_C, & \text{if } 0.0000 \le DANGCEMR_t < 0.0028 \\ S_D, & \text{if DANCEMR}_t \ge 0.0028 \end{cases} \quad 4.14$$

$$S_j = \begin{cases} S_A, & \text{if GTBR}_t < -0.0107 \\ S_B, & \text{if -0.0107} \le GTBR_t < 0.0000 \\ S_C, & \text{if } 0.0000 \le GTBR_t < 0.0121 \\ S_D, & \text{if GTBR}_t \ge 0.0121 \end{cases} \quad 4.15$$

Equations 4.13 to 4.15 are the stances in scenario 5, where the threshold applied was based on Quartiles (Q), and these classifications are as follows: $ASIR_t < -0.0047$, $DANGCEMR_t < -0.0022$ and $GTBR_t < -0.0107$ are classified as strong-low returns; $-0.0047 \le ASIR_t < 0.0000$, $-0.0022 \le DANGCEMR_t < 0.0000$ and $-0.0107 \le GTBR_t < 0.0000$ are classified as low returns; $0.0000 \le ASIR_t < 0.0048$, $0.0000 \le DANGCEMR_t < 0.0028$ and $0.0000 \le GTBR_t < 0.0121$ are classified as high returns; while $ASIR_t \ge 0.0048$, $DANGCEMR_t \ge 0.0028$ and $GTBR_t \ge 0.0121$ are classified as strong-high returns.

From scenario 2 to 5, the first research question; "What are the various states likely to identify for returns regime changes outside the traditional 2-state regime?", was adequately treated.

4.3 Frequency of states of returns

After classifying the respective states of returns, the result obtained was used to get the frequencies for each of the scenarios and for ASIR, DANGCEMR and GTBR, respectively. Tables 4.2 to 4.16 were extracted from Appendix 3.

Table 4.2: ASIR frequency of returns for two states (Scenario 1)

Category	Return State	Number of Days	Percentage
Low	< 0.0000	1,458	47.55
High	≥ 0.0000	1,608	52.45
Total		3,066	

Source: Produced by the Author.

Table 4.3: DANGCEMR frequency of returns for two states (Scenario 1)

Category	Return State	Number of Days	Percentage
Low return	< 0.0000	561	30.05
High return	≥ 0.0000	1,306	69.95
Total		1,867	

Source: Produced by the Author

Table 4.4: GTBR frequency of returns for two states (Scenario 1)

Category	Return State	Number of Days	Percentage
Low return	< 0.0000	1,423	46.41
High return	≥ 0.0000	1,643	53.59
Total		3,066	

Source: Produced by the Author

Tables 4.2, 4.3 and 4.4 give the frequency of returns for scenario 1, with two categories (named as low and high returns), for all the returns variables; ASIR, DANGCEMR and GTBR. The frequencies for high returns were higher than those of low returns. Since high returns indicate bullish period in the market and low return indicate bearish period, the results above support the standpoint of Yaya and Gil-Alana (2014) that the bull market (high returns) persist longer than the bear market.

Tables 4.5, 4.6 and 4.7 give the frequency of returns for scenario 2, with categories tagged as low, medium and high returns, across the board. The three tables reflect the evidence of moderate returns in the Nigeria stock market. If traders, investors and other market participants take a position or enter the market in an appropriate entry period, in terms of pricing; the probabilities of having a moderate return will be high for the three variables (ASIR, DANGCEMR and GTBR). Zhang and Zhang (2009) and Madhav 2017 used similar approach to obtain the probability of each state. The results are not comparable, since they are not of the same time frame.

Table 4.5: ASIR frequency of returns for three states (Scenario 2)

Category	Return State	Number of Days	Percentage
Low return	< -0.1164	4	0.13
Medium return	$[-0.1164 \quad 0.1167)$	3,058	99.74
High return	≥ 0.1167	4	0.13
Total			

Source: Produced by the Author

Table 4.6: DANGCEMR frequency of returns for three states (Scenario 2)

Category	Return State	Number of Days	Percentage
Low return	< -0.0197	188	10.07
Medium return	$[-0.0197 \quad 0.0203)$	1,500	80.34
High return	≥ 0.0203	179	9.59
Total		1,867	

Source: Produced by the Author

Table 4.7: GTBR frequency of returns for three states (Scenario 2)

Category	**Return State**	**Number of Days**	**Percentage**
Low return	< -0.0302	288	9.40
Medium return	$[-0.0302 \quad 0.0310)$	2,436	79.45
High return	≥ 0.0310	342	11.15
Total		3,066	

Source: Produced by the Author.

Tables 4.8, 4.9 and 4.10 give the frequency of returns for scenario 3, with categories tagged as low, medium and high returns, across the board. Here also, the three tables show the evidence of moderate returns in the Nigeria stock market. The traders, investors and other market participants have opportunities of making moderate returns of about 50% chances, provided they enter the market at an appropriate time, when the market is down and exit, when market is up. These chances of moderate returns were the same for the three variables (ASIR, DANGCEMR and GTBR).

Tables 4.11, 4.12 and 4.13 give the frequency of returns for scenario 4, with categories tagged as strong-low, low, high and strong-high returns, across the board. In this scenario, the chances for abnormal returns (positive or negative) are minimal. From the three tables, there is evidence of more low returns than high returns, in probability terms, for Nigeria stock market.

Tables 4.14, 4.15 and 4.16 give the frequency of returns for scenario 5, with categories tagged as strong-low, low, high and strong-high returns, across the board. In this scenario, apart from DANGCEMR that shows high returns, the other two variables (ASIR and GTBR) give marginally high returns of about 27.56% and 28.70%, respectively.

Table 4.8: ASIR frequency of returns for three states (Scenario 3)

Category	**Return State**	**Number of Days**	**Percentage**
Low return	< -0.0047	767	25.02
Medium return	$[-0.0047 \quad 0.0048)$	1,536	50.10
High return	≥ 0.0048	763	24.88
Total		3,066	

Source: Produced by the Author

Table 4.9: DANGCEMR frequency of returns for three states (Scenario 3)

Category	Return State	Number of Days	Percentage
Low return	< -0.0022	471	25.23
Medium return	$[-0.0022 \quad 0.0028)$	929	49.76
High return	≥ 0.0028	467	25.01
Total		1,867	

Source: Produced by the Author.

Table 4.10: GTBR frequency of returns for three states (Scenario 3)

Category	Return State	Number of Days	Percentage
Low return	< -0.0107	769	25.08
Medium return	$[-0.0107 \quad 0.0121)$	1,534	50.03
High return	≥ 0.0121	763	24.89
Total		3,066	

Source: Produced by the Author.

Scenario 4

Table 4.11: ASIR frequency of returns for four states (Scenario 4)

Category	**Return State**	**Number of Days**	**Percentage**
Strong-low return	< -0.1164	4	0.13
Low return	$[-0.1164 \quad 0.0002)$	1,720	56.10
High return	$[0.0002 \quad 0.1167)$	1,338	43.64
Strong-high return	≥ 0.1167	4	0.13
Total		3,066	

Source: Produced by the Author

Table 4.12: DANGCEMR frequency of returns for four states (Scenario 4)

Category	**Return State**	**Number of Days**	**Percentage**
Strong-low return	< -0.0197	188	10.07
Low return	$[-0.0197 \quad 0.0003)$	1,145	61.33
High return	$[0.0003 \quad 0.0203)$	355	19.01
Strong-high return	≥ 0.0203	179	9.59
Total		1,867	

Source: Produced by the Author

Table 4.13: GTBR frequency of returns for four states (Scenario 4)

Category	Return State	Number of Days	Percentage
Strong-low return	< -0.0302	288	9.40
Low return	$[-0.0302 \quad 0.0004)$	1,366	44.55
High return	$[0.0004 \quad 0.0310)$	1,070	34.90
Strong-high return	≥ 0.0310	342	11.15
Total		3,066	

Source: Produced by the Author.

Table 4.14: ASIR frequency of returns for four states (Scenario 5)

Category	Return State	Number of Days	Percentage
Strong-low return	< -0.0047	767	25.02
Low return	$[-0.0047 \quad 0.0000)$	691	22.54
High return	$[0.0000 \quad 0.0048)$	845	27.56
Strong-high return	≥ 0.0048	763	24.88
Total		3,066	

Source: Produced by the Author.

Table 4.15: DANGCEMR frequency of returns for four states (Scenario 5)

Category	Return State	Number of Days	Percentage
Strong-low return	< -0.0022	470	25.17
Low return	$[-0.0022 \quad 0.0000)$	92	4.93
High return	$[0.0000 \quad 0.0028)$	838	44.88
Strong-high return	≥ 0.0028	467	25.01
Total		1,867	

Source: Produced by the Author.

Table 4.16: GTBR frequency of returns for four states (Scenario 5)

Category	Return State	Number of Days	Percentage
Strong-low return	< -0.0107	769	25.08
Low return	$[-0.0107 \quad 0.0000)$	654	21.33
High return	$[0.0000 \quad 0.0121)$	880	28.70
Strong-high return	≥ 0.0121	763	24.89
Total		3,066	

Source: Produced by the Author.

4.4 Transition Probabilities

The transition probabilities, as on Tables 4.17 to 4.21 are extracted from the transition probability matrix of the daily returns of the Nigerian stock market returns (ASIR), Dangote Cement (DANGCEMR) and Guaranty Trust Bank (GTBR), in Appendix 3.

Scenario 1

According to the result in Table 4.17, the transition probability matrix indicates the ratio value for changes in the index/stock returns movements in two successive trading days. From the first scenario, which is a two-state transition matrix, it means that if the return on ASI is in the lower state on a particular day, the next day's return will be lower or higher with probabilities 0.5734 and 0.4266, respectively. Also, if it is high in a particular day, the probabilities that it will be low or high the next day are 0.3871 and 0.6129, respectively. For DANGCEM that is in low state of return on a particular day, the probabilities of the next day's low or high states are 0.3173 and 0.6827, respectively; while if it is in a high state on a particular day, the probabilities for the next day's states of low return or high return are 0.2965 and 0.7065, respectively. Similarly for GTB returns that is in low return on a particular day, the probabilities that it will be low or high the next day are 0.5074 and 0.4976, respectively; while if it is high on a particular day, the probability that it will be in low or high states of returns are 0.4269 and 0.5731, respectively.

Scenario 2

For the second scenario, as presented in Table 4.18, if the ASI daily's return is low on a particular day, the next day's return will be medium or high with probabilities 0.75 and 0.25, respectively. If it is in medium return, the next day's state of market of low, medium and high return will be 0.0007, 0.9987 and 0.0007, respectively. Also, if the daily's return state is in high return, probabilities of the following day's state of return to be low, medium and high return will be 0.5, 0.25 and 0.5, respectively.

Table 4.17: Transition Probabilities for Two State

Transition		ASIR	DANGCEMR	GTBR
Low Return to Low Return	p_{11}	**0.5734**	**0.3173**	**0.5074**
Low Return to High Return	p_{12}	0.4266	0.6827	0.4926
High Return to Low Return	p_{21}	0.3871	0.2965	0.4269
High Return to High Return	p_{22}	**0.6129**	**0.7065**	**0.5731**

Source: Extracted from Scenario 1 (ASIR, DANGCEMR and GTBR) in Appendix 3.

Table 4.18: Transition Probabilities for Three State

Transition		ASIR	DANGCEMR	GTBR
Low Return to Low Return	p_{11}	**0.0000**	**0.1330**	**0.2604**
Low Return to Medium Return	p_{12}	0.7500	0.7181	0.5660
Low Return to High Return	p_{13}	0.2500	0.1489	0.1736
Medium Return to Low Return	p_{21}	0.0007	0.0847	0.0690
Medium Return to Medium Return	p_{22}	**0.9987**	**0.8332**	**0.8575**
Medium Return to High Return	p_{23}	0.0007	0.0821	0.0735
High Return to Low Return	p_{31}	0.5000	0.2011	0.1316
High Return to Medium Return	p_{32}	0.2500	0.6425	0.5380
High Return to High Return	p_{33}	**0.2500**	**0.1564**	**0.3304**

Source: Extracted from Scenario 2 (ASIR, DANGCEMR and GTBR) in Appendix 3

If DANGCEM daily's return is in low return, the next day's return will be low, medium or high with probabilities of 0.1330, 0.7181 and 0.1489, respectively. Similarly, if it is in medium return state, the next day's state of market for low, medium and high return will be 0.0847, 0.8332 and 0.0821, respectively. Also, if the same DANGCEM daily's return state is in high return state, the probabilities of the following day's state of return to be low, medium and high return will be 0.2011, 0.6425 and 0.1564, respectively. Likewise, for GTB daily's return that is currently in low return, the next day's return will be in low, medium or high with probabilities 0.2604, 0.5660 and 0.1736, respectively. Similarly, for it to be currently positioned at the medium return state, the next day's state of market for low, medium and high return will be 0.0690, 0.8575 and 0.0735, respectively. Likewise, if GTBR daily's return state is in high return, the probabilities of the following day's state of return to be in low, medium and high returns, will be 0.1316, 0.5380 and 0.3304, respectively. If the entire market return (ASIR) is in medium return state, it returns to the same medium return state faster than in the cases of GTBR and DANGCEMR.

Scenario 3

With a clear disparity from scenario 2, the third scenario result is presented in Table 4.19. The ASI daily's return is in low return, will move to low, medium or high returns state by the next day with probabilities 0.4146, 0.4094 and 0.1760, respectively. If it is in medium return, the next day's state of market of low, medium and high return will be 0.2090, 0.5820 and 0.2090, respectively. Also, if the daily's return state is in high return, the following day's states of return to lower, medium and high return will be 0.1680, 0.4304 and 0.4016, respectively. When DANGCEM daily's return is in low return, the next day's return will be in low, medium or high with probabilities 0.2660, 0.4255 and 0.3085, respectively. Similarly, if it is in medium return, the following day's states of market for low, medium and high return will be 0.2196, 0.5813 and 0.1991, respectively. Also, if the same DANGCEM daily's return state is in high return, the following day's state of return to low, medium and high return will be 0.3019, 0.4026 and 0.2955, respectively.

Table 4.19: Transition Probabilities for Three State

Transition		ASIR	DANGCEMR	GTBR
Low Return to Low Return	p_{11}	**0.4146**	**0.2660**	**0.3394**
Low Return to Medium Return	p_{12}	0.4094	0.4255	0.4083
Low Return to High Return	p_{13}	0.1760	0.3085	0.2523
Medium Return to Low Return	p_{21}	0.2090	0.2196	0.2081
Medium Return to Medium Return	p_{22}	**0.5820**	**0.5813**	**0.6145**
Medium Return to High Return	p_{23}	0.2090	0.1991	0.1774
High Return to Low Return	p_{31}	0.1680	0.3019	0.2477
High Return to Medium Return	p_{32}	0.4304	0.4026	0.3644
High Return to High Return	p_{33}	**0.4016**	**0.2955**	**0.3879**

Source: Extracted from Scenario 3 (ASIR, DANGCEMR and GTBR) in Appendix 3

For GTB daily's return that is currently in low return, the next day's return will be in low, medium or high with probabilities 0.3394, 0.4083 and 0.2523, respectively. Similarly, for it to be currently position of medium return, the next day's state of market for low, medium and high returns will be 0.2081, 0.6145 and 0.1774, respectively. Likewise, if GTBR daily's return state is in high return, the following day's state of return to low, medium and high returns will be 0.2477, 0.3644 and 0.3879, respectively. If the entire market return (ASIR) is in medium return, it returns to the same medium return state faster than in the cases of GTBR and DANGCEMR.

Scenario 4

Here, the fourth scenario result is presented in Table 4.20. The ASI daily's return is in strong-low return is shown to move, by the next day to low, high and strong-high return state, with probabilities 0.5, 0.25 and 0.25, respectively. If it is in low return, the following day's state of market of strong-low, low, high and strong-high returns will be 0.0006, 0.6453, 0.3535 and 0.0006, respectively. Also, if the daily's return state is in high return, the following day's state of return to strong-low, low, high and strong-high returns will be 0.0007, 0.454, 0.5445 and 0.0007, respectively. Furthermore, if the daily's return state is in strong-high return, the following day's state of return to strong-low, low, high and strong-high return will be 0.5, 0.25, 0.0 and 0.5, respectively.

If DANGCEM daily's return is in strong-low return, the next day's return will be strong-low, low, high and strong-high with probabilities 0.1330, 0.5160, 0.2021 and 0.1489, respectively. If it is in low return, the next day's state of market of strong-low, low, high and strong-high return will be 0.0945, 0.6518, 0.1750 and 0.0787, respectively. Also, if the daily's return state is in high return, the following day's state of return to strong-low, low, high and strong-high return will be 0.0534, 0.6067, 0.2472 and 0.0927, respectively. Moreover, if the daily's return state is in strong-high return, the following day's state of return to strong-low, low, high and strong-high return will be 0.2011, 0.4693,0.1732 and 0.1564, respectively.

Table 4.20: Transition Probabilities for Four States

Transition		ASIR	DANGCEMR	GTBR
S/Low Return to S/Low Return	p_{11}	**0.0000**	**0.1330**	**0.2604**
S/Low Return to Low Return	p_{12}	0.5000	0.5160	0.2951
S/Low Return to High Return	p_{13}	0.2500	0.2021	0.2708
S/Low Return to S/High Return	p_{14}	0.2500	0.1489	0.1736
Low Return to S/Low Return	p_{21}	0.0006	0.0945	0.0886
Low Return to Low Return	p_{22}	**0.6453**	**0.6518**	**0.5007**
Low Return to High Return	p_{23}	0.3535	0.1750	0.3602
Low Return to S/High Return	p_{24}	0.0006	0.0787	0.0505
High Return to S/Low Return	p_{31}	0.0007	0.0534	0.0440
High Return to Low Return	p_{32}	0.4540	0.6067	0.4705
High Return to High Return	p_{33}	**0.5445**	**0.2472**	**0.3826**
High Return to S/High Return	p_{34}	0.0007	0.0927	0.1029
S/High Return to S/Low Return	p_{41}	0.5000	0.2011	0.1316
S/High Return to Lower Return	p_{42}	0.2500	0.4693	0.2749
S/High Return to High Return	p_{43}	0.0000	0.1732	0.2632
S/High Return to S/High Return	p_{44}	**0.2500**	**0.1564**	**0.3304**

Source: Extracted from Scenario 4 (ASIR, DANGCEMR and GTBR) in Appendix 3

For GTB daily's return that is currently in strong- low return, the next day's return will be strong-low, low, high and strong-high with probabilities 0.2604, 0.2951, 0.2708 and 0.1736, respectively. If it is in low return, the probabilities of the next day's state of market being in the strong-low, low, high and strong-high return states will be 0.0886, 0.5007, 0.3602 and 0.0505, respectively. When the daily's return state is in high return, the following day's state of return to strong-low, low, high and strong-high return will be with probabilities 0.0440, 0.4705, 0.3826, and 0.1029, respectively. Furthermore, if the daily's return state is in strong-high return, the probabilities that the following day's state of return will be in strong-low, low, high and strong-high return are 0.1316, 0.2749, 0.2632 and 0.3304, respectively.

Scenario 5

The fifth scenario result is presented in Table 4.21. ASI daily's return is in strong-low return is likely to move, by the next day, to strong-low, low, high and strong-high return state with probabilities 0.4146, 0.1864, 0.2229 and 0.1760, respectively. If it is in low return, the next day's state of market is likely to be in strong-low, low, high and strong- high return states with probabilities 0.2243, 0.3184, 0.2996 and 0.1577, respectively. Also, if the daily's return state is in high return, the following day's state of return is likely to be in strong-low, low, high and strong-high return states with probabilities 0.1964, 0.2402, 0.3124 and 0.2509, respectively. Furthermore, if the daily's return state is in strong-high return, the following day's state of return will be in strong-low, low, high and strong-high return states with probabilities 0.1680, 0.1640, 0.2664 and 0.4016, respectively.

If DANGCEM daily's return is in strong-low return, the next day's return will be in strong-low, low, high and strong-high return states with probabilities 0.2660, 0.0511, 0.3745 and 0.3085, respectively. If it is in low return, the following day's state of market is likely to be in strong-low, low, high and strong-high return state with probabilities 0.2527, 0.0659, 0.5165 and 0.1648, respectively. If the daily's return state is in high return, the following day's state of return is likely to move to strong-low, low, high and strong-high return with probabilities 0.2160, 0.0549, 0.5263 and 0.2029, respectively. If the daily's return state is however in strong-high return, the following day's state of return may be strong-low, low, high and strong-high return with probabilities 0.3019, 0.0321, 0.3704 and 0.2955, respectively.

For GTB daily's return that is currently in strong-low return, the next day's return will be strong-low, low, high and strong-high with probabilities 0.3394, 0.1678, 0.2406 and 0.2523, respectively. If it is in low return, the following day's state of market will be strong-low, low, high and strong- high return with probabilities 0.2385, 0.2691, 0.3440 and 0.1483, respectively. When the daily's return state is in high return, the following day's state of return are likely to be strong-low, low, high and strong-high return with probabilities 0.1854, 0.2594, 0.3561 and 0.1991, respectively. Furthermore, if the daily's return state is in strong-high return, the following day's state of return will be strong-low, low, high and strong-high return with probabilities 0.2477, 0.1586, 0.2058 and 0.3879, respectively.

The calculations for transition probabilities as shown on Tables 4.17 to 4.21 for scenarios 1 to 5, have addressed research question two, "What will be the forecast, in the probability terms, of the returns of the stock market whose future value is influenced only by its current state and not any prior activity that may lead the return to its current position?"

Table 4.21: Transition Probabilities for Four States

Transition		ASIR	DANGCEMR	GTBR
S/Low Return to S/Low Return	p_{11}	**0.4146**	**0.2660**	**0.3394**
S/Low Return to Low Return	p_{12}	0.1864	0.0511	0.1678
S/Low Return to High Return	p_{13}	0.2229	0.3745	0.2406
S/Low Return to S/High Return	p_{14}	0.1760	0.3085	0.2523
Low Return to S/Low Return	p_{21}	0.2243	0.2527	0.2385
Low Return to Low Return	p_{22}	**0.3184**	**0.0659**	**0.2691**
Low Return to High Return	p_{23}	0.2996	0.5165	0.3440
Low Return to S/High Return	p_{24}	0.1577	0.1648	0.1483
High Return to S/Low Return	p_{31}	0.1964	0.2160	0.1854
High Return to Low Return	p_{32}	0.2402	0.0549	0.2594
High Return to High Return	p_{33}	**0.3124**	**0.5263**	**0.3561**
High Return to S/High Return	p_{34}	0.2509	0.2029	0.1991
S/High Return to S/Low Return	p_{41}	0.1680	0.3019	0.2477
S/High Return to Low Return	p_{42}	0.1640	0.0321	0.1586
S/High Return to High Return	p_{43}	0.2664	0.3704	0.2058
S/High Return to S/High Return	p_{44}	**0.4016**	**0.2955**	**0.3879**

Source: Extracted from Scenario 5 (ASIR, DANGCEMR and GTBR) in Appendix 3

4.5 Test of Independence

The stochastic variables sequences do have a Markovian property, which is necessary for the analysis of Markov chain model and in calculating the transition probabilities using χ^2. The test hypotheses that establish the Markovian property are given as;

H_0: Successive transitions are independent (current state and next state are independent)

H_1: Successive transitions are not independent (current state and next state are dependent).

Table 4.22 shows the calculations of the test for Independence;

All the p-values are less than the stated level, $(\alpha = 0.05)$, with the exception a 2-state for Dangcem. Imperatively, the null that the successive transitions are not independent cannot be rejected at 5% level, which implies that the current and the next states are dependent. This Markov property tests conducted indicates that the returns follow a Markov chain of first-order, and by implication, a day's return relies only on its immediate previous day's return. This has fulfilled the Markovian property.

Table 4.22: Chi-Square Computation

		χ^2	Degrees of freedom	p-value
Scenario 1	ASIR	105.67	1	$2.2e^{-16}$
	DANGCEMR	**0.9471**	**1**	**0.3304**
	GTBR	19.52	1	$9.9e^{-06}$
Scenario 2	ASIR	1145.9	4	$2.2e^{-16}$
	DANGCEMR	48.136	4	$8.84e^{-10}$
	GTBR	353.97	4	$2.2e^{-16}$
Scenario 3	ASIR	252.07	4	$2.2e^{-16}$
	DANGCEMR	55.173	4	$2.98e^{-11}$
	GTBR	207.21	4	$2.2e^{-16}$
Scenario 4	ASIR	1258.2	9	$2.2e^{-16}$
	DANGCEMR	63.699	9	$2.582e^{-10}$
	GTBR	386.04	9	$2.2e^{-16}$
Scenario 5	ASIR	284.73	9	$2.2e^{-16}$
	DANGCEMR	127.89	9	$2.2e^{-16}$
	GTBR	216.01	9	$2.2e^{-16}$

Source: Produced by the Author

4.6 Long-Run Behavior of the Market Returns

Scenario 1

From Table 4.23, the limiting probability (steady-state) vector for the first scenario show that λ(ASIR) = (0.4757, 0.5243); which implies that 47.57% of the time, the return of the index will transit to a low state, while 52.43% of the time, it will transit to a high state. For λ(DANGCEMR) = (0.3007, 0.6993); in approximately 30.07% of the time, the returns on DANGCEMR will move to a low state, while in 69.93% of the time, it transits to a high state. For λ(GTBR) = (0.4643, 0.5357); in about 46.43% of the time, the returns on GTB will change to a low state, while 53.57% of the time, it will transit to a high state. In all cases, a steady-state point of four (4) trading days is achieved, which implies that the market will stabilize, with corresponding stated probabilities, on the fourth day of continuous trading.

Scenario 2

The long-run probability of the market returns for second scenario result is presented in Table 4.24. It can be observed that λ(ASIR) = (0.0014, 0.9972, 0.0014); which indicates that in approximately 0.14% of the time, the index will change to a low state, in 99.72% of the time to medium state, while 0.14% of the time, it transits to a high state. With λ(DANGCEMR) = (0.1007, 0.8033, 0.0960), DANGCEM transits to a low state in approximately 10.07% of the time, while in 80.33% of the time, it changes to medium, and in 9.60% of the time, it will change to high state of return. Similarly, λ(GTBR) = (0.0940, 0.7944, 0.1116) means that in 9.40% of the time, the returns on GTB will move to low return; in 79.44% of the time, it will move to medium return; while in about 11.16% of the time, it transits to high return.

Table 4.23: Long-run Probability of the Market Returns (Scenario 1)

	Point of Limiting n	$\lambda_i = (\lambda_1, \lambda_2)$
ASIR	4	(0.4757,0.5243)
DANGCEMR	4	(0.3007, 0.6993)
GTBR	4	(0.4643, 0.5357)

$\lambda_1 - low\ return,\ \lambda_2 - high\ return$

Table 4.24: Long-run Probability of the Market Returns (Scenario 2)

	Point of Limiting n	$\lambda_t = (\lambda_1,\ \lambda_2,\ \lambda_3)$
ASIR	15	(0.0014,0.9972,0.0014)
DANGCEMR	4	(0.1007,0.8033,0.0960)
GTBR	8	(0.0940,0.7944,0.1116)

$\lambda_1 - low\ return,\ \lambda_2 - medium\ return,\ \lambda_3 - high\ return$

Scenario 3

For the third scenario, the long-run probability of the market returns is presented in Table 4.25. From the observed result, λ(ASIR) = (0.2503, 0.5011,0.2486), which implies that in 25.03% of the time ASIR state of return will change to low state; in about 50.11% of the time, it will change to medium state; while in about 24.86% of the time, it will change to high state. With λ(DANGCEMR) = (0.2519, 0.4972, 0.2508), in about 25.19% of the time, returns for DANGCEMR will change to low state; in about 49.72% of the time, it moves to medium state; and in about 25.08% of the time, it will shift to high state of return. For λ(GTBR) = (0.2509, 0.5006, 0.2485), in about 25.09% of the time, the return state moves to low; in about 50.06% of the time, it moves to medium; while in about 24.85% of the time, the returns moves to high state.

Scenario 4

In the fourth scenario, the limiting distribution is presented in Table 4.26 and shows that λ(ASIR) = (0.0013, 0.5609, 0.4360, 0.0013), which implies that in approximately 0.1% of the time, ASIR state of return will change to strong-low state; in approximately 56.09% of the time, it will change to low state; in approximately 43.60% of the time, it will be in high state; while in approximately 0.1% of the time it will change to strong-high. On the DAGCEM returns, the limiting distribution is given by λ(DANGCEMR) = (0.1007, 0.6120, 0.1914, 0.0959), which implies that in 10.07% of the time, the state of the return will change to strong-low; in about 61.20% of the time, the state of returns will change to low; in approximately 19.14% of the time, the state of the return will change to high; while in approximately 9.59% of the time, the return will change to strong-high. Similarly, for the GTBR, the obtained limiting distribution is λ(GTBR) = (0.0940, 0.4457, 0.3488, 0.1116), which indicates that in about 9.40% of the time, GTBR state of return will change to strong-low; in about 44.57% of the time, the state of return will change to low state; in about 34.88% of the time, the state of return will change to high return; while in approximately 11.16% of the time, the state of return will change to strong-high.

Table 4.25: Long-run Probability of the Market Returns (Scenario 3)

	Point of Limiting n	$\lambda_t = (\lambda_1,\ \lambda_2,\ \lambda_3)$
ASIR	8	(0.2503, 0.5011, 0.2486)
DANGCEMR	6	(0.2519, 0.4972, 0.2508)
GTBR	7	(0.2509, 0.5006,0.2485)

$\lambda_1 - low\ return,\ \lambda_2 - medium\ return,\ \lambda_3 - high\ return$

Table 4.26: Long-run Probability of the Market Returns (Scenario 4)

	Point of Limiting n	$\lambda_t = (\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4)$
ASIR	15	(0.0013, 0.5609, 0.4360, 0.0013)
DANGCEMR	6	(0.1007, 0.6120, 0.1914, 0.0959)
GTBR	9	(0.0940, 0.4457, 0.3488, 0.1116)

$\lambda_1 - strong\ low\ return,\ \lambda_2 - low\ return,\ \lambda_3 - high\ return,\ \lambda_4 - strong\ high\ return$

Scenario 5

From Table 4.27, the limiting distribution of the fifth scenario is presented. For ASIR, the obtained limiting distribution is λ(ASIR) = (0.2501, 0.2253, 0.2755, 0.2485), which indicates that in approximately 25.01% of the time, ASIR state of return will change to strong-low state; in approximately 22.53% of the time, it will change to low state; in approximately 27.55% of the time, it will be in high state; while in 24.85% of the time, it will change to strong-high. The limiting distribution obtained for DANGCEM is λ(DANGCEMR) = (0.2520, 0.0488, 0.4486, 0.2509), which indicates that in about 25.20% of the time, the state of the return will change to strong-low; in about 4.88% of the time, the state of returns will change to lower; in approximately 44.86% of the time, the state of the return will change to high; while in about 25.09% of the time, the return will change to strong-high. Likewise in the case of GTBR, the limiting distribution is obtained as λ(GTBR) = (0.2508, 0.2134, 0.2872, 0.2485) and indicates that in about 25.08% of the time, GTBR state of return will change to strong-low; in about 21.34% of the time, the state of return will change to low state; in approximately 28.72% of the time, the state of return will change to high return; while in about 24.85% of the time, the state of return will change to strong-high.

Tables 4.23 to 4.27, which indicate the limiting distribution and point of limiting point of limiting distributions, have solved both research questions 3 and 4 together, as the question stated thus; “What will be the various probability distributions which remain unchanged after various stages of transition as time progress and similarly what will be the stable probabilities for each of the state of market returns?” and “At what time will the market returns for each of the regimes reach a stable point?”

Table 4.27: Long-run Probability of the Market Returns (Scenario 5)

	Point of Limiting n	$\lambda_t = (\lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4)$
ASIR	8	(0.2501, 0.2253, 0.2755, 0.2485)
DANGCEMR	7	(0.2520, 0.0488, 0.4486, 0.2509)
GTBR	9	(0.2508, 0.2134, 0.2872, 0.2485)

$\lambda_1 - strong\ low\ return,\ \lambda_2 - low\ return,\ \lambda_3 - high\ return,\ \lambda_4 - strong\ high\ return$

4.7 Computation of Mean Return Times of the Markov Chain

Scenario 1

From Table 4.28, the mean return times for low and high returns are as follows: For ASIR, the obtained mean returns is 2 days; for DANGCEMR, it lies between 2 and 3 days; while for GTBR, the mean return time is between 1 and 2 days. ASIR returns faster to previous position, followed by GTBR, while it takes a longer time for DANGCEMR to return to any of its previous position.

Scenario 2

Table 4.29 shows that the mean return time was over predicted for scenario 2. For ASIR, it will take about 716 days to return to low return, just 1 day in returning to medium return, and about 716 days to return to high return position. However, the case is different for DANGCEMR, as it takes about 10 days to return low return, 1 day to return to medium return and 10 days to return to high return. Also, for GTBR, it will take approximately 10 days to return to low return, 1 day to return to medium return and 9 days to return to high return.

Scenario 3

Table 4.30 shows that the mean return time gives a moderate timing of returns for scenario 3, we have as follows; for ASIR it will take 4 days to return low return, and 2 days in returning to medium return while also takes 4 days to return to high return position. More like that of ASIR, it shows that for DANGCEMR it takes 4 days to return low return, 2 days to return to medium return and 4 days to return to high return. Also for GTBR, it will take 4 days for returning to low return, 2 days to return to medium return and 4 days to return to high-return.

Table 4.28: Mean Return Times of the Market Returns (Scenario 1)

Variables	$\mu_t = (\mu_1,\ \mu_2)$
ASIR	(2.10, 1.90)
DANGCEMR	(3.32, 1.42)
GTBR	(2.15, 1.86)

$\mu_1 - low\ return,\ \ \mu_2 - high\ return$

Table 4.29: Mean Return Times of the Market Returns (Scenario 2)

Variables	$\mu_t = (\mu_1,\ \mu_2,\ \mu_3)$
ASIR	(716.42, 1.00, 716.42)
DANGCEMR	(9.92, 1.24, 10.42)
GTBR	(10.64, 1.25, 8.96)

$\mu_1 - low\ return,\ \mu_2 - medium\ return,\ \mu_3 - high\ return$

Table 4.30: Mean Return Times of the Market Returns (Scenario 3)

Variables	$\mu_t = (\mu_1,\ \mu_2,\ \mu_3)$
ASIR	(3.99, 1.99, 4.02)
DANGCEMR	(3.99, 2.01,3.98)
GTBR	(3.99, 1.99, 4.02)

$\mu_1 - low\ return,\ \ \mu_2 - medium\ return,\ \ \mu_3 - high\ return$

Scenario 4

Table 4.31 presents the mean return time for scenario 4. For ASIR, it will take approximately 778 days to return to strong-low return position, 2 days to return to low return position, 2 days to return to high return position and 778 days to return to strong high return position. However, the case is different for DANGCEMR, as it takes about 10 days to return strong low return position, 2 days to return to low return position, 5 days to return to high return position and 10 days in returning to strong high return position. Similarly for GTBR, it will take 10 days for GTBR to return to strong-low return position, 2 days to return to low return position, 3 days to return to high return position and 9 days to return to strong high return position.

Scenario 5

Table 4.32 presents the mean returns for the fifth scenario. It is observed that for ASIR, it will take approximately 4 days to return strong-low return, 4 days to return to low return position, 3 days to return to high return position and 4 days to return to strong-high return position. Also, for DANGCEMR, it takes 4 days to return strong-low return position, 2 days to return to low return position, 2 days to return to high return position and 4 days in returning to strong high return position. In the case of GTBR, it will take about 4 days for GTBR to return to strong low return position, approximately 5 days to return to low return position, about 3 days to return to high return position and approximately 4 days to return to strong high return position.

Table 4.31: Mean Return Times of the Market Returns (Scenario 4)

Variables	$\mu_t = (\mu_1, \mu_2, \mu_3, \mu_4)$
ASIR	(778.68,1.78,2.29,778.64)
DANGCEMR	(9.92, 1.63, 5.22,10.42)
GTBR	(10.64, 2.24,2.86,8.96)

$\mu_1 - strong\ low\ return,\ \mu_2 - low\ return,\ \mu_3 - high\ return,\ \mu_4 - strong\ high\ return$

Table 4.32: Mean Return Times of the Market Returns (Scenario 5)

Variables	$\mu_i = (\mu_1,\ \mu_2,\ \mu_3,\ \mu_4)$
ASIR	(3.99, 4.43, 3.60, 4.02)
DANGCEMR	(3.96, 2.05, 2.22, 3.98)
GTBR	(3.99, 4.68, 3.48, 4.02)

$\mu_1 - strong\ low\ return,\ \mu_2 - low\ return,\ \mu_3 - high\ return,\ \mu_4 - strong\ high\ return$

4.8 Computation of Occupancy Times

Scenario 1

From Tables 4.33 and 4.34, two cases are considered, which include when $n=5$ and $n=10$, and they represent the trading days for the expected amount of time spent during transition period from low return to low return. The number of days for ASIR and GTBR greater than that of DANGCEMR. The case is, however, different for transition of high return to high return. DANGCEMR is found to remain in high- to- high (4 days) than the likes of ASIR and GTBR. For the 3 variables under study, the cross interstate returns remain in low returns between 1 and 2 days for transition from low to high and high to low. This indicates that DANGCEM is more stable in giving high returns to the investors than GTB and the entire market index (ASIR). When $n=10$, ASIR and GTBR reported above 5 days, staying in low return to low return. While for high-to-high, DANGCEMR could conveniently stay for 8 out of 10 days, in compensating the investors. It has high period of consistence for high returns.

Scenario 2

From Tables 4.35 and 4.36, when $n=5$; from low-to-low returns, all the three variables trend along the same line, maintaining less than 2 days within the low transition state. Similarly for medium to medium, all reported the same number of days, except ASIR that is very close to 6 days (overestimated). Transition from low to medium returns is also high for ASIR, DANGCEMR and GTBR. Similarly, transition from medium to high return was also high, while the transition from high to high was low across the board. In the same way, for $n=10$ days, all the three variables remained on low to low transition states for less than 2 days; for the low to medium return transition states, occupancy period for ASIR was higher compared to those of DANGCEMR and GTBR. In this scenario, the occupancy period was also overestimated for the case of ASIR (estimates of above 10 days), while DANGCEMR and GTBR were close to 10 days. The transitions from high to high returns remained between 1 to 2 days across the board, having occupancy period of above 8 days for ASIR to make a reversal transit from high to medium return, and above 7 days for DANGCEMR and GTBR each.

Table 4.33: Occupancy Times for Two States (Scenario 1) - $n = 5$

Transition		ASIR	DANGCEMR	GTBR
Low Return to Low Return	m_{11}	**3.4986**	**2.5203**	**3.3682**
Low Return to High Return	m_{12}	2.5013	3.4796	2.6317
High Return to Low Return	m_{21}	2.2697	1.4959	2.2807
High Return to High Return	m_{22}	**3.7302**	**4.5040**	**3.7192**

Table 4.34: Occupancy Times for Two States (Scenario 1) - $n = 10$

Transition		ASIR	DANGCEMR	GTBR
Low Return to Low Return	m_{11}	**5.8773**	**4.0236**	**5.6896**
Low Return to High Return	m_{12}	5.1226	6.9763	5.3103
High Return to Low Return	m_{21}	4.6483	2.9992	4.4602
High Return to High Return	m_{22}	**6.3516**	**8.0008**	**6.3979**

Table 4.35: Occupancy Times of the Market Returns-Three States (Scenario 2) - $n = 5$

Transition		ASIR	DANGCEMR	GTBR
Low Return to Low Return	m_{11}	**1.1930**	**1.5449**	**1.6840**
Low Return to Medium Return	m_{12}	4.4144	3.9151	3.6462
Low Return to High Return	m_{13}	0.3932	0.5399	0.6696
Medium Return to Low Return	m_{21}	0.0056	0.4848	0.4347
Medium Return to Medium Return	m_{22}	**5.9902**	**5.0517**	**5.0632**
Medium Return to High Return	m_{23}	0.0056	0.4634	0.5020
High Return to Low Return	m_{31}	0.7819	0.6181	0.5395
High Return to Medium Return	m_{32}	3.6268	3.8285	3.5994
High Return to High Return	m_{33}	**1.5817**	**1.5533**	**1.8610**

Table 4.36: Occupancy Times of the Market Returns-Three States (Scenario 2) - $n = 10$

Transition		ASIR	DANGCEMR	GTBR
Low Return to Low Return	m_{11}	**1.2100**	**2.0486**	**2.1542**
Low Return to Medium Return	m_{12}	9.3839	7.9316	7.6177
Low Return to High Return	m_{13}	0.4100	1.0197	1.2280
Medium Return to Low Return	m_{21}	0.0125	0.9885	0.9044
Medium Return to Medium Return	m_{22}	**10.9803**	**9.0682**	**9.0358**
Medium Return to High Return	m_{23}	0.0125	0.9432	1.0597
High Return to Low Return	m_{31}	0.8085	1.1218	1.0097
High Return to Medium Return	m_{32}	8.5861	7.8450	7.5706
High Return to High Return	m_{33}	**1.6085**	**2.0331**	**2.4196**

Scenario 3

In scenario 3, (results presented in Tables 4.37 and 4.38), the occupancy time reported during the transition period from low to low returns is approximately within 2 days each for ASIR, DANGEMR and GTBR, when $n = 5$. Similarly, the occupancy time was in the same trend for the transition period from low to medium return, while transition period from low to high returns and medium to low returns maintained the bound of 1 day, across the board. In this scenario also, the market was always stable during the transition from medium to medium returns for a period of 3 days and above for ASIR, DANGCEMR and GTBR. The cross state of returns movement from medium to high returns and from high to low returns' occupancy times was above 1 day for all the variables, while the transition returns for high to high returns revolves around 2 days for ASIR, DANGCEM and GTBR.

A situation of having $n = 10$, the occupancy time reported during the transition period for low to low returns was above 3 days for ASIR, DANGEMR and GTBR. The trend in the occupancy time the same for the transition period from low to medium return, which is also above 4 days; while transition period from low to high returns and medium to low returns maintained the domain of 2 days for all the variables. When $n = 5$, the market is always stable during the transition from medium to medium returns for a period of about 6 days and above for ASIR, DANGCEMR and GTBR. The cross state of returns movement from medium to high returns and from high to low returns' occupancy time is above 2 days for all the variables, while the transition returns for high to high returns was above 3 days for ASIR, DANGCEM and GTBR.

Scenario 4

In scenario 4 result on occupancy time reported in Tables 4.39 and 4.40, during the transition period for strong/low to strong/low returns was above 1 day for ASIR, DANGEMR and GTBR when $n = 5$. The occupancy time showed similar trends for the transition period from strong/low to low return of above 2 days, while transition period from strong/low to high returns was within 1 day bound, and strong/low to strong/high returns maintained less than 1 day across the board. In this scenario, market was always stable during the transition from low to low returns, for a period of above 3 days, for ASIR, DANGCEMR and GTBR. The cross state of returns movement from high to low returns' occupancy times was above 3 days for all the variables.

Table 4.37: Occupancy Times of the Market Returns-Three States (Scenario 3) - $n = 5$

Transition		ASIR	DANGCEMR	GTBR
Low Return to Low Return	m_{11}	**2.4631**	**2.2801**	**2.3569**
Low Return to Medium Return	m_{12}	2.3922	2.3993	2.3860
Low Return to High Return	m_{13}	1.1446	1.3204	1.2569
Medium Return to Low Return	m_{21}	1.2017	1.2204	1.2008
Medium Return to Medium Return	m_{22}	**3.6021**	**3.5873**	**3.6517**
Medium Return to High Return	m_{23}	1.1960	1.1922	1.1474
High Return to Low Return	m_{31}	1.1378	1.3169	1.2587
High Return to Medium Return	m_{32}	2.4250	2.3725	2.3218
High Return to High Return	m_{33}	**2.4371**	**2.3105**	**2.4194**

Table 4.38: Occupancy Times of the Market Returns-Three States (Scenario 3) - $n = 10$

Transition		ASIR	DANGCEMR	GTBR
Low Return to Low Return	m_{11}	**3.7145**	**3.5398**	**3.6113**
Low Return to Medium Return	m_{12}	4.8977	4.8854	4.8890
Low Return to High Return	m_{13}	2.3876	2.5747	2.4995
Medium Return to Low Return	m_{21}	2.4530	2.4800	2.4552
Medium Return to Medium Return	m_{22}	**6.1076**	**6.0735**	**6.1549**
Medium Return to High Return	m_{23}	2.4392	2.4464	2.3898
High Return to Low Return	m_{31}	2.3890	2.5766	2.5132
High Return to Medium Return	m_{32}	4.9305	4.8586	4.8247
High Return to High Return	m_{33}	**3.6803**	**3.5647**	**3.6619**

Table 4.39: Occupancy Times of the Market Returns-Four States (Scenario 4) - $n = 5$

Transition		ASIR	DANGCEMR	GTBR
S/Low Return to S/Low Return	m_{11}	**1.1927**	**1.5434**	**1.6824**
S/Low Return to Low Return	m_{12}	2.6464	2.9469	2.0118
S/Low Return to High Return	m_{13}	1.7675	0.9694	1.6334
S/Low Return to S/High Return	m_{14}	0.3929	0.5401	0.6716
Low Return to S/Low Return	m_{21}	0.0050	0.4957	0.4579
Low Return to Low Return	m_{22}	**3.9098**	**4.1051**	**3.3042**
Low Return to High Return	m_{23}	2.0795	0.9390	1.7650
Low Return to S/High Return	m_{24}	0.0050	0.4599	0.4727
High Return to S/Low Return	m_{31}	0.0053	0.4512	0.4063
High Return to Low Return	m_{32}	2.6731	3.0603	2.2692
High Return to High Return	m_{33}	**3.3152**	**2.0155**	**2.7862**
High Return to S/High Return	m_{34}	0.0053	0.4729	0.5381
S/High Return to S/Low Return	m_{41}	0.7817	0.6169	0.5376
S/High Return to Low Return	m_{42}	2.3264	2.8891	1.9790
S/High Return to High Return	m_{43}	1.3101	0.9404	1.6202
S/High Return to S/High Return	m_{44}	**1.5815**	**1.5534**	**1.8636**

Table 4.40: Occupancy Times of the Market Returns-Four States (Scenario 4) - $n=10$

Transition		ASIR	DANGCEMR	GTBR
S/Low Return to S/Low Return	m_{11}	**1.2091**	**2.0471**	**2.1526**
S/Low Return to Low Return	m_{12}	5.4464	6.0068	4.2392
S/Low Return to High Return	m_{13}	3.9336	1.9263	3.3769
S/Low Return to S/High Return	m_{14}	0.4091	1.0197	1.2301
Low Return to S/Low Return	m_{21}	0.0114	0.9994	0.9278
Low Return to Low Return	m_{22}	**6.7149**	**7.1651**	**5.5326**
Low Return to High Return	m_{23}	4.2602	1.8959	3.5091
Low Return to S/High Return	m_{24}	0.0114	0.9395	1.0304
High Return to S/Low Return	m_{31}	0.0117	0.9548	0.8762
High Return to Low Return	m_{32}	5.4777	6.1202	4.4976
High Return to High Return	m_{33}	**5.4957**	**2.9723**	**4.5302**
High Return to S/High Return	m_{34}	0.0117	0.9524	1.0959
S/High Return to S/Low Return	m_{41}	0.8077	1.1206	1.0080
S/High Return to Low Return	m_{42}	5.1215	5.9490	4.2068
S/High Return to High Return	m_{43}	3.4616	1.8973	3.3641
S/High Return to S/High Return	m_{44}	**1.6077**	**2.0329**	**2.4223**

Also, the transition return for high to high returns was above 3 days for ASIR, while it was above 2 days for both DANGCEM and GTBR. The transition returns for strong/high to strong/high returns was above 2 days for ASIR, DANGCEM and GTBR.

For $n=10$, the occupancy time reported during the transition period for strong/low to strong/low returns was within 2 days for ASIR, DANGEMR and GTBR. The occupancy time was between 4 and 6 days for the transition period from strong/low to low returns, while the transition period from strong/low to high returns are the same for ASIR and GTBR, with above 3 days, while for DANGCEMR, it was below 2 days. Ditto for $n=5$, as market is always stable during the transition from low to low returns, for a period of 5 days and above for ASIR, DANGCEMR and GTBR. The transition occupancy time for high to high returns was above 5 days for ASIR, and above 3 days for both DANGCEM and GTBR. The cross state of returns movement from high to low returns' occupancy times was above 4 days for all the variables. The transition returns for strong/high to strong/high returns was greater than 1 day for ASIR, and above 2 days for both DANGCEM and GTBR.

Scenario 5

From Tables 4.40 and 4.41, the occupancy time reported during the transition period for strong/ low to strong/low returns was above 2 days for ASIR, DANGEMR and GTBR when $n=5$. However, the occupancy time was not similar for the transition period from strong/low to low return, as it was above 1 day for ASIR and GTBR, and less than 1 day for DANGCEMR. The transition period from strong/low to high returns followed the same pattern, as ASIR and GTBR were above 1 day, while DANGCEMR was above 2 days. Strong/low to strong/high returns maintained above 1 day across the board. In this scenario, the market was less stable during the transition from low to low returns for a period of between 1 and 2 days for ASIR, DANGCEMR and GTBR. The cross state of returns movement from high to low returns' occupancy times was above 1 day for ASIR and GTBR, while it was less than 1 day for DANGCEMR. The transition returns for high to high returns was above 3 days for DAMGCEMR, and above 2 days for ASIR and GTBR, which is contrary to what was obtained in Scenario 4. The transition returns for strong/high to strong/high returns was above 2 days for ASIR, DANGCEM and GTBR.

For $n = 10$, the occupancy time reported during the transition period for strong/low to strong/low returns was above 3 days for ASIR, DANGEMR and GTBR. However, the occupancy time was not the same for the transition period from strong/low to low return, as it was above 2 days for ASIR and GTBR, and less than 1 day for DANGCEMR. The transition period from strong/low to high returns followed the same pattern, as ASIR and GTBR was above 2 days, while DANGCEMR was above 4 days. Strong-low to strong-high returns maintained above 2 days across the board. In this scenario, market was less stable during the transition from low to low returns for a period of between 1 and 3 days for ASIR, DANGCEMR and GTBR. The cross state of returns movement from high to low returns' occupancy times was above 2 days for ASIR and GTBR, and less than 1 day for DANGCEMR. The transition returns for high to high returns was greater than 5 days for DAMGCEMR, greater than 3 days for both ASIR and GTBR. This also contradicts the stance in Scenario 4. The transition returns for strong/high to strong/high returns was greater than 3 days for ASIR, DANGCEM and GTBR.

Following the computed occupancy times, which was the expected time spent at the various states of DTMC, as obtained from Tables 4.33 to 4.42, research question 5 has been adequately addressed.

Table 4.41: Occupancy Times of the Market Returns-Four States (Scenario 5) - $n = 5$

Transition		ASIR	DANGCEMR	GTBR
S/Low Return to S/Low Return	m_{11}	**2.4630**	**2.2806**	**2.3565**
S/Low Return to Low Return	m_{12}	1.0804	0.2445	1.0102
S/Low Return to High Return	m_{13}	1.3116	2.1552	1.3761
S/Low Return to S/High Return	m_{14}	1.1438	1.3204	1.2576
Low Return to S/Low Return	m_{21}	1.2261	1.2524	1.2345
Low Return to Low Return	m_{22}	**2.2390**	**1.2633**	**2.1382**
Low Return to High Return	m_{23}	1.4084	2.3262	1.5134
Low Return to S/High Return	m_{24}	1.1259	1.1578	1.1133
High Return to S/Low Return	m_{31}	1.1826	1.2170	1.1755
High Return to Low Return	m_{32}	1.1464	0.2515	1.1279
High Return to High Return	m_{33}	**2.4204**	**3.3359**	**2.5236**
High Return to S/High Return	m_{34}	1.2494	1.1963	1.1727
S/High Return to S/Low Return	m_{41}	1.1361	1.3166	1.2588
S/High Return to Low Return	m_{42}	1.0507	0.2254	0.9934
S/High Return to High Return	m_{43}	1.3711	2.1469	1.3284
S/High Return to S/High Return	m_{44}	**2.4415**	**2.3109**	**2.4192**

Table 4.42: Occupancy Times of the Market Returns-Four States (Scenario 5) - $n = 10$

Transition		ASIR	DANGCEMR	GTBR
S/Low Return to S/Low Return	m_{11}	**3.7137**	**3.5407**	**3.6109**
S/Low Return to Low Return	m_{12}	2.2070	0.4884	2.0776
S/Low Return to High Return	m_{13}	2.6893	4.3983	2.8123
S/Low Return to S/High Return	m_{14}	2.3863	2.5752	2.5004
Low Return to S/Low Return	m_{21}	2.4769	2.5123	2.4886
Low Return to Low Return	m_{22}	**3.3657**	**1.5072**	**3.2054**
Low Return to High Return	m_{23}	2.7863	4.5689	2.9493
Low Return to S/High Return	m_{24}	2.3685	2.4124	2.3557
High Return to S/Low Return	m_{31}	2.4332	2.4771	2.4297
High Return to Low Return	m_{32}	2.2730	0.4954	2.1952
High Return to High Return	m_{33}	**3.7983**	**5.5791**	**3.9596**
High Return to S/High Return	m_{34}	2.4920	2.4511	2.4153
S/High Return to S/Low Return	m_{41}	2.3867	2.5764	2.5131
S/High Return to Low Return	m_{42}	2.1774	0.4692	2.0606
S/High Return to High Return	m_{43}	2.7491	4.3895	2.7643
S/High Return to S/High Return	m_{44}	**3.6843**	**3.5654**	**3.6620**

4.9 Discussion of Results

In this study, Microsoft Excel package was used for the sorting and classification of various states, before proceeding with further analysis using R codes, specifically written for the estimation of the transition probabilities, limiting distributions, mean return times, the initial state probabilities and occupancy times.

The transition probability matrix specifies the ratio value for changes in the index/stock return movements in two successive trading days. From the first scenario, which is two-state transition matrix, ASI return is in lower state is likely to transit to lower state with probability 0.5734 or to a higher state with probability 0.4266. It could also transit from a high return state low state with probability 0.3871 or a high return state with probability 0.6129. DANGCEM returns can transit from a low state to another low state with probability 0.3173 or to a high return state with probability 0.6827. From a high return state, it could transit to a low return state with probability 0.2965 or to a high return state with probability 0.7065. With respect to GTB returns transiting from low return state to a low and high return state, it does so with probabilities 0.5074 and 0.4976, respectively, and for a transition from high to low and high return state, it moves with probabilities 0.4269 and 0.5731, respectively.

For the second scenario, the ASI daily's return transits from a lower return state to medium and high return states with respective probabilities 0.75 and 0.25. It transition from medium return state to lower, medium and higher return state are with probabilities 0.0007, 0.9987 and 0.0007, respectively. From higher return state to lower, medium and higher return states, the transition probabilities are 0.5, 0.25 and 0.5, respectively.

4.9.1 Limiting Distribution (Equilibrium)

The model does not provide forecasting results in absolute form (Madhav, 2017). The initial state vector and the transition probability matrices are used in estimating the probability of ASIR, DANGCEMR and GTBR, being in different states in the upcoming days.

From the steady-state, the limiting probability vector in the first scenario given as λ(ASIR) = (0.4757, 0.5243) implies that 47.57% of the period of the index return will transit to a low state, while 52.43% of the time to a high state. For λ(DANGCEMR) = (0.3007, 0.6993) means that 30.07% of the time the DANGCEMR's return will move to a low state, while 69.93% of the time to a high state. And λ(GTBR) = (0.4643, 0.5357) implies that 46.43% of the time, the GTB's return will change to a low state, while 53.57% of the time to a high state.

From the second scenario, λ(ASIR) = (0.0014, 0.9972, 0.0014) indicates that 0.14% of the period of the index return will change to a low state, 99.72% of the time to medium state, while 0.14% of the time to a high state. With λ(DANGCEMR) = (0.1007, 0.8033, 0.0960) indicates that 10.07% of the time, the state of return will change to low state, 80.33% of the time it will change to medium, while 9.60% of the time it will change to high state of return. Similarly λ(GTBR) = (0.0940, 0.7944, 0.1116) means that 9.40% of the time the GTB's return will move to low return, 79.44% of the time it will move to medium return, while at 11.16% of the time to high return.

On the third scenario, λ(ASIR) = (0.2503, 0.5011,0.2486) shows that 25.03% of the time ASIR state of return will change to low state, 50.11% of the time it will change to medium state, while at 24.86% of the time it will change to high state. With λ(DANGCEMR) = (0.2519, 0.4972, 0.2508) means that 25.19% of the time returns for DANGCEM change to low state, 49.72% moves to medium state and 25.08% of the time shift to high state of return. For λ(GTBR) = (0.2509, 0.5006, 0.2485), 25.09% of the time the return state move to low, 50.06% of the time it moves to medium, while 24.85% of the time the return moves to high state.

The fourth scenario reveals that the limiting distribution is λ(ASIR) = (0.0013, 0.5609, 0.4360, 0.0013) and implies in 0.1% of the time ASIR state of return will change to strong- low state, 56.09% of the time it will change to low state and 43.60% of the time it will be in high state, and 0.1% of the time it will change to strong-high. The limiting distribution for λ(DANGCEMR) = (0.1007, 0.6120, 0.1914, 0.0959) also indicates that 10.07% of the time the state of the return will change to strong-low, 61.20% of the time the state of returns will change to low, 19.14% of the time the state of the return will change to high, while 9.59% of the time the return will change to strong-high. Also, for the GTBR will obtained the limiting distribution λ(GTBR) = (0.0940, 0.4457, 0.3488, 0.1116), which indicate that 9.40% of the time GTB state of return will change to strong-low, 44.57% of the time the state of return will change to low state, 34.88% of the time the state of return will change to high return while 11.16% of the time the state of return will change to strong-high.

For the fifth scenario, the obtained limiting distribution was λ (ASIR) = (0.2501, 0.2253, 0.2755, 0.2485), which predicted that 25.01% of the time ASIR state of return will change to strong-low state, 22.53% of the time it will change to low state and 27.55% of the time it will be in high state and 24.85% of the time it will change to strong-high. The limiting distribution λ (DANGCEMR) = (0.2520, 0.0488, 0.4486, 0.2509) indicates that 25.20% of the time the state of the return will change to strong-low, 4.88% of the time the state of returns will change to lower, 44.86% of the time the state of the return will change to high, while 25.09% of the time the return will change to strong-high. Likewise we have the limiting distribution λ (GTBR) = (0.2508, 0.2134, 0.2872, 0.2485) which indicate that 25.08% of the time GTBR state of return will change to strong-low, 21.34% of the time the state of return will change to low state, 28.72% of the time the state of return will change to high return, while 24.85% of the time the state of return will change to strong-high.

4.9.2 Mean Return Times

From scenario 1, the mean return times for low return and high return were as follows: ASIR (2.10, 1.90); DANGCEMR (3.32, 1.42); GTBR (2.18, 1.86). The mean returns time was over predicted in scenario 2, we have as follows; ASIR (716.42, 1.0, 716.42); DANGCEMR (9.92, 1.24, 10.42); GTBR (10.64, 1.25, 8.96), for low-return, medium and high-return, respectively.

From scenario 3, also have as follows: ASIR (3.99, 1.99, 4.02); DANGCEMR (3.99, 2.01, 3.98); GTBR (3.98, 2.0, 4.02), for low-return, medium and high-return, respectively.

Scenario 4 gives an over predicting the mean return times as follows: ASIR (778.68, 1.78, 2.29, 778.64); DANGCEMR (9.92, 1.63, 5.22, 10.42); GTBR (10.64, 2.24, 2.86, 8.96), for strong-low, low, high and strong-high return, respectively.

For scenario 5, the mean return times are as follows: ASIR (3.99, 4.43, 3.6, 4.02); DANGCEMR (3.96, 2.05, 2.22, 3.98); GTBR (3.99, 4.68, 3.48, 4.02); for strong-low, low, high and strong-high returns, respectively.

4.9.3 Occupancy Times

Results on the expected amount of time spent in a given interval of time during the transition period for each the states reveal that in scenario 1, between 2 to 4 days out of 5 days were spent on a particular state transitions, and between 3 to 8 days out of 10 days across the three series.

For Scenario 2, the expected amount of time spent during the transition period was between 0 to 6 days out of 5 days and between 0 to 11 days out of 10 days, an over-estimated value of the occupancy times noted when the mean and standard deviation was used as a threshold for the classification of the states.

In scenario 3, the expected amount of time spent during the transition period was between 2 and 3 days out 5 days, while it was between 4 and 6 days out of 10 days. Similarly, in scenario 5, the expected amount of time spent to between 2 and 3 out of 5 days and between 2 and 6 out of 10 days.

The existing literatures that applied the same procedure for the transition probabilities, limiting distributions and expected returns are as follows; Zhang and Zhang (2009), Madhav (2017), Idolor et.at. (2018), Doubleday and Esung (2011), and Vasanthi Subha and Nambi (2011). As they deferred from this research study in terms of time horizon, mostly focused on monthly and weekly data and none of them go further to look at the occupancy times of the states. Okonta et.al (2017) that was fairly similar to this study also used weekly data and considered only two states of returns.

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Summary of Findings

In this work, the impact of the stock market returns on the global economy as it rises and falls was investigated, with focus on the global market and the Nigeria stock market returns. Also, the study was motivated by the need to work on the probability and conditional probability of Nigerian market returns, as it moves from one transition to the other. Evidence of state transition in the Nigerian stock market returns was also established into strong-low, low, high and strong-high.

The decision to adapt MCMs to the Nigerian stock market returns was duly justified for the study. Similarly, the gap identified was well addressed with the extension of 2-state to 3-state and 4-state of market returns in Nigerian stock market. The daily frequency data was used, as against weekly data used by Okonta et al. (2017). Following this, persistence of returns was noticed on the state transition probability. Stationary probabilities, expected time of revisit to the states and expected time spent on each state transition were adequately fitted in the Nigeria stock market returns.

The choice of MCMs for this study was further reinforced by Zhang and Zhang (2009) describing Markov Chain prediction methods as a probability forecasting method. The predicted results simply expressed the probability of a certain state of stock returns in the future, rather than in the absolute state. Markov prediction therefore plays an important role in modern day statistics because of its Markovian properties of "no after effect". There is a weak demand on historical data. The difference between Markov Model and other statistical methods (such as time series and regression analysis) is that it does not need to find mutual laws among the factors from the predictor. It simply considers the characteristics of the evolution on the history situation of the event itself and calculates changes of the internal classified states.

Considering the various risks associated with investment in stocks, investors and financial portfolio managers need to follow the transition probabilities of various state of returns, identified in this research, as the effects of those risk are signals to the changes in the probabilities of the transition. Having looked at the All share Index, Dangcem and GTB, over the stated sample period, the study reveals that market returns comprise of one of two, three and four different states of transition, at the close of each trading day. The trend of the market is either upward or downward (bull or bear) for 2-state; up, down or zero (bull, bear and stagnant states) for 3-state; and strong up, up, down and strong down (bull, retracement, support and bear states). This could be likened to the economic cycle that has four stages (expansion, peak, contraction and trough). Noting very clearly during the expansion state; the economy would experience a rapid development, growth in business coupled with low interest rate and increase in production. During contraction, there is recession and correction of the economy. The period of contraction is characterized by slow growth, fall in employment, while prices of goods and services remain stagnant. The trough period is when economy is at a low point of growth, from where the recovery takes off.

On the major deductions, the contributions of extreme returns are noted at both ends (lower side and upper side). Expected time spent on each states of more than 2 days in a week and 5 days in two weeks would allow the investors and traders enough time to take decision, either to enter the market or exit from the market. The estimated revisit time of the state gives the reason for short term trading opportunity in the market of selling high and buying low, or selling low and buying lower, within a short interval of revisits of each state.

5.2 Contributions to Knowledge

Extension of the return to 3-state and 4-state with the use of mean, standard deviation and quartiles as the thresholds for the classification of the states of returns across the three series. Using the MCMs of 3-state and 4-state in predicting the Nigerian Stock Market return series.

Here, probabilities statements about the 2-state, 3-state and 4-state of the returns were made for the All Share Index, Dangote Cement Plc and Guaranty Trust Bank Plc.

The increase in the limiting and expected return times in days for scenarios 4 and 5 is good for an investor, as it allows more room for investment before return to equilibrium. It further supports the classification of valuation and recommendation of stocks into; strong-sell, sell, buy and strong-buy.

The occupancy time in a state is the time spent in that state, over that system's life time; this was a contribution as previous researches in stock market returns are unable to introduce it to determine the time spent in life time of a state of market returns' life time. Models based on absorbing Markov Chains provide a powerful framework for the analysis of occupancy times as mentioned by Kulkani (2011)

5.3 Limitation of the Study

Non availability of hourly data during the period of this research was a major limitation, as the intra-day transition of the state of returns and the prediction of the regime changes within the trading days in Nigerian Stock Market could not be captured.

5.4 Areas of Future Research

Several studies on share price movements and returns of the Nigeria stock markets are available in extant literature, including specifically works of scholars like Davou et al. (2013), Afolabi and Dada (2014) and Okonta et al. (2017). However, most of the studies focus on the first tier arm of the market, while the second tier remain un-touched, and also, with more attention on the ASI. Further studies can be spread over the following sectorial indices:

1. NSE Banking Sector Index
2. NSE Industrial Goods Index
3. NSE Consumer Goods Index
4. NSE Oil and Gas Index
5. NSE Insurance Index.

More future research can be carried out in applied MCMs to the five sectorial indices, in order to establish which sectorial index returns transition move along with the NSE ASIR, and also to predict the change pattern of each of these sectorial index returns.

Also, the following new customized indices exist: NSE 30 index, NSE Premium Index, NSE Pension Index, NSE Asem Index, NSE-AFR Bank Value Index, NSE-AFR Div Yield Index, NSE MERI Growth Index, NSE MERI Value Index and NSE Lotus II. New areas of future research could also be extended to increase in the memory of MCMs, by looking at 2nd order Markov chain process. Okonta et al. (2017) used weekly index, while this research used daily index. Further studies can be extended to the hourly index, to examine the intra-day probability transition of the returns.

5.5 Conclusions and Recommendations.

In summary, the equilibriums of the MCM showed that it will take a longer period (15 days) for the regimes to change, for scenarios 2 to 5. Persistence in state returns were well pronounced across the three series in the transition probabilities for scenarios 2 to 5, with occupancy times predicted at lower time (1.19 days) in scenario 3 and 5. This creates more liquidity in the market; and aligns with the findings of Chen and Hill (2013) that assert the existence of a close association between liquidity and returns, and allow a more frequent bargain, which is likely to attract more traders, stakeholders and participant. It will make the market more active and robust. The extended classification based on quartiles generally outperformed the hitherto 2-state regime classification and those of standard deviation. Therefore, regime changes in Nigerian Stock Market return are better predicted using Markov chain models based on the quartile classification.

Poorly performing stock market returns is an indication that investors are losing value; otherwise, the investors are gaining value. A high probability in the transition probabilities in favour of high- returns and strong-high returns states indicate that the stakeholders in the market are gaining value, while low transition probabilities reveal that the investors/traders participating in the market are losing value.

Therefore, in view of the above, the application of the use of MCM in determining the direction of the stock market returns will empower investors, traders and other market participants, like institutional investors; making a reasonable decision in minimizing losses and maximizing profits.

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APPENDIX 1

```
#  2 States  NSE Returns 2006 to 2018
library(expm)
library(markovchain)
library(diagram)
library(pracma)
stationdistTPM <- function(M){
 eigenprob <- eigen(t(M))
 temp <- which(round(eigenprob$values,1)==1)
 stationdist <- eigenprob$vectors[,temp]
 stationdist <- stationdist/sum(stationdist)
 return(stationdist)
 }
stateNames <- c("Lower","Higher")  # NSE Returns 2006 to 2018
P <- matrix(c(0.5734,0.4266,0.3871,0.6129),
nrow=2, byrow=TRUE)
row.names(P) <- stateNames; colnames(P) <- stateNames
P
 stationdistTPM(P)
1/stationdistTPM(P)
P0<- P%^% 0
P2<- P%^% 2
P3<- P%^% 3
P4<- P%^% 4
P5<- P%^% 5
P6<- P%^% 6
P7<- P%^% 7
P8<- P%^% 8
P9<- P%^% 9
P10<- P%^% 10
P12<- P%^% 12
P15<- P%^%15
# NSE Returns 2006 to 2018
round(P0, 4)
round(P2, 4)
round(P3, 4)
round(P4,4)
round(P5, 4)
round(P6, 4)
round(P7, 4)
round(P8,4)
round(P9, 4)
round(P10,4)
round(P12,4)
round(P15,4)
```

```
Poccup10=P0+P+P2+P3+P4+P5+P6+P7+P8+P9+P10
Poccup5=P0+P+P2+P3+P4+P5
Poccup10
Poccup5
plotmat(round(P,2),pos = c(1,1),
lwd = 1, box.lwd = 2,
cex.txt = 0.8,
box.size = 0.1,
box.type = "circle",
box.prop = 0.5,
box.col = "light yellow",
arr.length=.1,
arr.width=.1,
self.cex = .4,
self.shifty = -.01,
self.shiftx = .13,
main = "ASI Returns Markov Chain-(2006-2018)")
P3 <- P %^% 3
round(P3,4)
InitialStates <- c(1/2, 1/2)
round(InitialStates %*% P3,4)


#  3 States  NSE Returns 2006 to 2018


library(expm)
library(markovchain)
library(diagram)
library(pracma)


stationdistTPM <- function(M){
 eigenprob <- eigen(t(M))
 temp <- which(round(eigenprob$values,1)==1)
 stationdist <- eigenprob$vectors[,temp]
 stationdist <- stationdist/sum(stationdist)
 return(stationdist)
 }

stateNames <- c("Lower","Moderate","Higher")  # NSE Returns 2006 to 2018
P <- matrix(c(0.0,0.75,0.25,0.0007,0.9987,0.0007,0.5,0.25,0.25),
nrow=3, byrow=TRUE)
row.names(P) <- stateNames; colnames(P) <- stateNames

round(P,4)
```

```
stationdistTPM(P)

1/stationdistTPM(P)

P0<- P%^% 0
P2<- P%^% 2
P3<- P%^% 3
P4<- P%^% 4
P5<- P%^% 5
P6<- P%^% 6
P7<- P%^% 7
P8<- P%^% 8
P9<- P%^% 9
P10<- P%^% 10
P12<- P%^% 12
P15<- P%^%15

# NSE Returns 2006 to 2018

round(P0, 4)
round(P2, 4)
round(P3, 4)
round(P4,4)
round(P5, 4)
round(P6, 4)
round(P7, 4)
round(P8,4)
round(P9, 4)
round(P10,4)
round(P12,4)
round(P15,4)

Poccup10=P0+P+P2+P3+P4+P5+P6+P7+P8+P9+P10

Poccup5=P0+P+P2+P3+P4+P5

Poccup10
Poccup5

plotmat(round(P,2),pos = c(1,2),
lwd = 1, box.lwd = 2,
cex.txt = 0.8,
box.size = 0.1,
box.type = "circle",
box.prop = 0.5,
```

```
box.col = "light yellow",
arr.length=.1,
arr.width=.1,
self.cex = .4,
self.shifty = -.01,
self.shiftx = .13,
main = "ASI Returns Markov Chain-(2006-2018)")
P3 <- P %^% 3
round(P3,4)
InitialStates <- c(1/3, 1/3, 1/3)
round(InitialStates %*% P3,4)

#  3 States  NSE Returns 2006 to 2018

library(expm)
library(markovchain)
library(diagram)
library(pracma)


stationdistTPM <- function(M){
 eigenprob <- eigen(t(M))
 temp <- which(round(eigenprob$values,1)==1)
 stationdist <- eigenprob$vectors[,temp]
 stationdist <- stationdist/sum(stationdist)
 return(stationdist)
 }

stateNames <- c("Lower","Moderate","Higher")  # NSE Returns 2006 to 2018
P <- matrix(c(0.4146,0.4094,0.1760,0.2090,0.5820,0.2090,0.1680,0.4304,0.4016),
nrow=3, byrow=TRUE)
row.names(P) <- stateNames; colnames(P) <- stateNames

P

 stationdistTPM(P)

1/stationdistTPM(P)

P0<- P%^% 0
P2<- P%^% 2
P3<- P%^% 3
P4<- P%^% 4
P5<- P%^% 5
P6<- P%^% 6
P7<- P%^% 7
```

```
P8<- P%^% 8
P9<- P%^% 9
P10<- P%^% 10
P12<- P%^% 12
P15<- P%^%15

# NSE Returns 2006 to 2018

round(P0, 4)
round(P2, 4)
round(P3, 4)
round(P4,4)
round(P5, 4)
round(P6, 4)
round(P7, 4)
round(P8,4)
round(P9, 4)
round(P10,4)
round(P12,4)
round(P15,4)

Poccup10=P0+P+P2+P3+P4+P5+P6+P7+P8+P9+P10

Poccup5=P0+P+P2+P3+P4+P5

Poccup10
Poccup5

plotmat(round(P,2),pos = c(1,2),
lwd = 1, box.lwd = 2,
cex.txt = 0.8,
box.size = 0.1,
box.type = "circle",
box.prop = 0.5,
box.col = "light yellow",
arr.length=.1,
arr.width=.1,
self.cex = .4,
self.shifty = -.01,
self.shiftx = .13,
main = "ASI Returns Markov Chain-(2006-2018)")
P3 <- P %^% 3
round(P3,4)

InitialStates <- c(1/3, 1/3, 1/3)
round(InitialStates %*% P3,4)
```

```
#  4 States  NSE Returns 2006 to 2018

library(expm)
library(markovchain)
library(diagram)
library(pracma)


stationdistTPM <- function(M){
 eigenprob <- eigen(t(M))
 temp <- which(round(eigenprob$values,1)==1)
 stationdist <- eigenprob$vectors[,temp]
 stationdist <- stationdist/sum(stationdist)
 return(stationdist)
 }

stateNames <- c("StongLower","Lower","Higher","StrongHigher")  # NSE Returns 2006 to 2018
P                                                                                          <-
matrix(c(0.0,0.5,0.25,0.25,0.0006,0.6453,0.3535,0.0006,0.0007,0.4540,0.5445,0.0007,0.5,0.25,0.
0,0.25),
nrow=4, byrow=TRUE)
row.names(P) <- stateNames; colnames(P) <- stateNames

P

 stationdistTPM(P)

1/stationdistTPM(P)

P0<- P%^% 0
P2<- P%^% 2
P3<- P%^% 3
P4<- P%^% 4
P5<- P%^% 5
P6<- P%^% 6
P7<- P%^% 7
P8<- P%^% 8
P9<- P%^% 9
P10<- P%^% 10
P12<- P%^% 12
P15<- P%^%15

# NSE Returns 2006 to 2018

round(P0, 4)
```

```
round(P2, 4)
round(P3, 4)
round(P4,4)
round(P5, 4)
round(P6, 4)
round(P7, 4)
round(P8,4)
round(P9, 4)
round(P10,4)
round(P12,4)
round(P15,4)

Poccup10=P0+P+P2+P3+P4+P5+P6+P7+P8+P9+P10

Poccup5=P0+P+P2+P3+P4+P5

Poccup10
Poccup5

plotmat(round(P,2),pos = c(2,2),
lwd = 1, box.lwd = 2,
cex.txt = 0.8,
box.size = 0.1,
box.type = "circle",
box.prop = 0.5,
box.col = "light yellow",
arr.length=.1,
arr.width=.1,
self.cex = .4,
self.shifty = -.01,
self.shiftx = .13,
main = "ASI Returns Markov Chain-(2006-2018)")
P3 <- P %^% 3
round(P3,4)

InitialStates <- c(1/4, 1/4, 1/4,1/4)
round(InitialStates %*% P3,4)
#  4 States  NSE Returns 2006 to 2018

library(expm)
library(markovchain)
library(diagram)
library(pracma)


stationdistTPM <- function(M){
```

```
 eigenprob <- eigen(t(M))
 temp <- which(round(eigenprob$values,1)==1)
 stationdist <- eigenprob$vectors[,temp]
 stationdist <- stationdist/sum(stationdist)
 return(stationdist)
 }

stateNames <- c("StongLower","Lower","Higher","StrongHigher")  # NSE Returns 2006 to 2018
P <-
matrix(c(0.4146,0.1864,0.2229,0.1760,0.2243,0.3184,0.2996,0.1577,0.1964,0.2402,0.3124,0.250
9,0.1680,0.1640,0.2664,0.4016),
nrow=4, byrow=TRUE)
row.names(P) <- stateNames; colnames(P) <- stateNames

P

 stationdistTPM(P)

1/stationdistTPM(P)

P0<- P%^% 0
P2<- P%^% 2
P3<- P%^% 3
P4<- P%^% 4
P5<- P%^% 5
P6<- P%^% 6
P7<- P%^% 7
P8<- P%^% 8
P9<- P%^% 9
P10<- P%^% 10
P12<- P%^% 12
P15<- P%^%15

# NSE Returns 2006 to 2018

round(P0, 4)
round(P2, 4)
round(P3, 4)
round(P4,4)
round(P5, 4)
round(P6, 4)
round(P7, 4)
round(P8,4)
round(P9, 4)
round(P10,4)
round(P12,4)
```

```
round(P15,4)

Poccup10=P0+P+P2+P3+P4+P5+P6+P7+P8+P9+P10

Poccup5=P0+P+P2+P3+P4+P5

Poccup10
Poccup5

plotmat(round(P,2),pos = c(2,2),
lwd = 1, box.lwd = 2,
cex.txt = 0.8,
box.size = 0.1,
box.type = "circle",
box.prop = 0.5,
box.col = "light yellow",
arr.length=.1,
arr.width=.1,
self.cex = .4,
self.shifty = -.01,
self.shiftx = .13,
main = "ASI Returns Markov Chain-(2006-2018)")
P3 <- P %^% 3
round(P3,4)

InitialStates <- c(1/4, 1/4, 1/4,1/4)
round(InitialStates %*% P3,4)
```

APPENDIX 2

Computation of Chi-square
All Share Index Returns (ASIR)
Scenario 1

```
library (matrix)
library (MASS)
# table1 2006-2018
table1=matrix(c(836,622,622,985),ncol=2,byrow=TRUE)
colnames(table1)=c("NegativeReturn","PostiveReturn")
rownames(table1)=c("NegativeReturn","PostiveReturn")
# table1 2006-2018
table1
# table1 2006-2018
 chisq.test(table1)
```

Scenario 2

```
library (matrix)
library (MASS)
# table2 2006-2018
table2=matrix(c(0,3,1,2,3053,2,2,1,1),ncol=3,byrow=TRUE)
colnames(table2)=c("LowerReturn","ModerateReturn","HigherReturn")
rownames(table2)=c("LowerReturn","ModerateReturn","HigherReturn")
# table2 2006-2018
table2
# table2 2006-2018
 chisq.test(table2)
```

Scenario 3

```
library (matrix)
library (MASS)
# table3 2006-2018

table3=matrix(c(318,314,135,321,894,321,128,328,306),ncol=3,byrow=TRUE)
colnames(table3)=c("LowerReturn","ModerateReturn","HigherReturn")
rownames(table3)=c("LowerReturn","ModerateReturn","HigherReturn")
# table3 2006-2018
table3
# table3 2006-2018

 chisq.test(table3)
```

Scenario 4

```
library (matrix)
library (MASS)
# table4 2006-2018
table4=matrix(c(0,2,1,1,1,1110,608,1,1,607,728,1,2,1,0,1),ncol=4,byrow=TRUE)
```

```
colnames(table4)=c("SLowerReturn","LowerReturn","HigherReturn","SHigherReturn")
rownames(table4)=c("SLowerReturn","LowerReturn","HigherReturn","SHigherReturn")
# table4 2006-2018
table4

# table4 2006-2018
 chisq.test(table4)
Scenario 5
library (matrix)
library (MASS)
# table5 2006-2018
table5=matrix(c(318,143,171,135,155,220,207,109,166,203,264,212,128,125,203,306),ncol=4,b
yrow=TRUE)
colnames(table5)=c("SLowerReturn","LowerReturn","HigherReturn","SHigherReturn")
rownames(table5)=c("SLowerReturn","LowerReturn","HigherReturn","SHigherReturn")
# table5 2006-2018
table5
# table5 2006-2018
 chisq.test(table5)

DANGCEMR
Scenario 1
library (matrix)
library (MASS)
# table11 2006-2018
table11=matrix(c(178,383,383,922),ncol=2,byrow=TRUE)
colnames(table11)=c("NegativeReturn","PostiveReturn")
rownames(table11)=c("NegativeReturn","PostiveReturn")
# table11 2006-2018
table11
# table11 2006-2018

 chisq.test(table11)
Scenario 2
library (matrix)
library (MASS)
# table12 2006-2018

table12=matrix(c(25,135,28,127,1249,123,36,115,28),ncol=3,byrow=TRUE)
colnames(table12)=c("LowerReturn","ModerateReturn","HigherReturn")
rownames(table12)=c("LowerReturn","ModerateReturn","HigherReturn")
# table12 2006-2018
table12
# table12 2006-2018
 chisq.test(table12)
Scenario 3
```

```
library (matrix)
library (MASS)
# table13 2006-2018
table13=matrix(c(125,200,145,204,540,185,141,188,138),ncol=3,byrow=TRUE)
colnames(table13)=c("LowerReturn","ModerateReturn","HigherReturn")
rownames(table13)=c("LowerReturn","ModerateReturn","HigherReturn")
# table13 2006-2018
table13
# table13 2006-2018
 chisq.test(table13)
```

Scenario 4

```
library (matrix)
library (MASS)
#table14 2006-2018
table14=matrix(c(25,97,38,28,108,745,200,90,19,216,88,33,36,84,31,28),ncol=4,byrow=TRUE)
colnames(table14)=c("SLowerReturn","LowerReturn","HigherReturn","SHigherReturn")
rownames(table14)=c("SLowerReturn","LowerReturn","HigherReturn","SHigherReturn")
# table14 2006-2018
table14
# table14 2006-2018
 chisq.test(table14)
```

Scenario 5

```
library (matrix)
library (MASS)
# table15 2006-2018
table15=matrix(c(125,24,176,145,23,6,147,15,181,46,441,170,141,15,173,138),ncol=4,byrow=T
RUE)
colnames(table15)=c("SLowerReturn","LowerReturn","HigherReturn","SHigherReturn")
rownames(table15)=c("SLowerReturn","LowerReturn","HigherReturn","SHigherReturn")
# table15 2006-2018
table15
# table15 2006-2018
 chisq.test(table15)
```

GTBR

Scenario 1

```
library (matrix)
library (MASS)
# table6 2006-2018
table6=matrix(c(722,701,701,941),ncol=2,byrow=TRUE)
colnames(table6)=c("NegativeReturn","PostiveReturn")
rownames(table6)=c("NegativeReturn","PostiveReturn")
# table6 2006-2018
table6
# table6 2006-2018
 chisq.test(table6)
```

Scenario 2

```
library (matrix)
library (MASS)

# table7 2006-2018
table7=matrix(c(75,163,50,168,2088,179,45,184,113),ncol=3,byrow=TRUE)
colnames(table7)=c("LowerReturn","ModerateReturn","HigherReturn")
rownames(table7)=c("LowerReturn","ModerateReturn","HigherReturn")
# table7 2006-2018
table7
# table7 2006-2018
 chisq.test(table7)
```

Scenario 3

```
library (matrix)
library (MASS)
# table8 2006-2018
table8=matrix(c(261,314,194,319,942,272,189,278,296),ncol=3,byrow=TRUE)
colnames(table8)=c("LowerReturn","ModerateReturn","HigherReturn")
rownames(table8)=c("LowerReturn","ModerateReturn","HigherReturn")
# table8 2006-2018
table8
# table8 2006-2018
 chisq.test(table8)
```

Scenario 4

```
 library (matrix)
library (MASS)
# table9 2006-2018
table9=matrix(c(75,85,78,50,121,684,492,69,47,503,409,110,45,94,90,113),ncol=4,byrow=TRU
E)
colnames(table9)=c("SLowerReturn","LowerReturn","HigherReturn","SHigherReturn")
rownames(table9)=c("SLowerReturn","LowerReturn","HigherReturn","SHigherReturn")
# table9 2006-2018
table9
# table9 2006-2018
 chisq.test(table9)
```

Scenario 5

```
library (matrix)
library (MASS)
# table10 2006-2018
table10=matrix(c(261,129,185,194,156,176,225,97,163,228,313,175,189,121,157,296),ncol=4,b
yrow=TRUE)
colnames(table10)=c("SLowerReturn","LowerReturn","HigherReturn","SHigherReturn")
rownames(table10)=c("SLowerReturn","LowerReturn","HigherReturn","SHigherReturn")
# table10 2006-2018
table10
# table10 2006-2018
 chisq.test(table10)
```

APPENDIX 3

Scenario 1

Markov Frequencies Matrixes for the Transition from States i to j

$$n_{ij(ASIR)} = \begin{pmatrix} 836 & 622 \\ 622 & 986 \end{pmatrix} \qquad P_{ij(ASIR)} = \begin{pmatrix} 0.5734 & 0.4266 \\ 0.3871 & 0.6129 \end{pmatrix}$$

$$n_{ij(\text{DANGCEM}R)} = \begin{pmatrix} 178 & 383 \\ 383 & 922 \end{pmatrix} \qquad P_{ij(\text{DANGCEM}R)} = \begin{pmatrix} 0.3173 & 0.6827 \\ 0.2935 & 0.7065 \end{pmatrix}$$

$$n_{ij(\text{GTB}R)} = \begin{pmatrix} 722 & 701 \\ 701 & 941 \end{pmatrix} \qquad P_{ij(\text{GTB}R)} = \begin{pmatrix} 0.5074 & 0.4926 \\ 0.4269 & 0.5731 \end{pmatrix}$$

Scenario 2

Markov Frequencies Matrixes for the Transition Occurrence from States i to j

$$n_{ij(ASIR)} = \begin{pmatrix} 0 & 3 & 1 \\ 2 & 3054 & 2 \\ 2 & 1 & 1 \end{pmatrix} \qquad P_{ij(ASIR)} = \begin{pmatrix} 0.0000 & 0.75 & 0.25 \\ 0.0007 & 0.9987 & 0.0007 \\ 0.50 & 0.25 & 0.25 \end{pmatrix}$$

$$n_{ij(\text{DANGCEM}R)} = \begin{pmatrix} 25 & 135 & 28 \\ 127 & 1249 & 123 \\ 36 & 115 & 28 \end{pmatrix} \qquad P_{ij(\text{DANGCEM}R)} = \begin{pmatrix} 0.1330 & 0.7181 & 0.1489 \\ 0.0847 & 0.8332 & 0.0821 \\ 0.2011 & 0.6425 & 0.1564 \end{pmatrix}$$

$$n_{ij(\mathrm{GTB}R)} = \begin{pmatrix} 75 & 163 & 50 \\ 168 & 2088 & 179 \\ 45 & 184 & 113 \end{pmatrix} \qquad P_{ij(\mathrm{GTB}R)} = \begin{pmatrix} 0.2604 & 0.5660 & 0.1736 \\ 0.0690 & 0.8575 & 0.0735 \\ 0.1316 & 0.5380 & 0.3304 \end{pmatrix}$$

Scenario 3

Markov Frequencies Matrixes for the Transition Occurrence from States i to j

$$n_{ij(ASIR)} = \begin{pmatrix} 318 & 314 & 135 \\ 321 & 894 & 321 \\ 128 & 328 & 307 \end{pmatrix} \qquad P_{ij(ASIR)} = \begin{pmatrix} 0.4146 & 0.4094 & 0.1760 \\ 0.2090 & 0.5820 & 0.2090 \\ 0.1680 & 0.4304 & 0.4016 \end{pmatrix}$$

$$n_{ij(\mathrm{DANGCEM}R)} = \begin{pmatrix} 125 & 200 & 145 \\ 204 & 540 & 185 \\ 141 & 188 & 138 \end{pmatrix} \qquad P_{ij(\mathrm{DANGCEM}R)} = \begin{pmatrix} 0.2660 & 0.4255 & 0.3085 \\ 0.2196 & 0.5813 & 0.1991 \\ 0.3019 & 0.4026 & 0.2955 \end{pmatrix}$$

$$n_{ij(\mathrm{GTB}R)} = \begin{pmatrix} 261 & 314 & 194 \\ 319 & 942 & 272 \\ 189 & 278 & 296 \end{pmatrix} \qquad P_{ij(\mathrm{GTB}R)} = \begin{pmatrix} 0.3394 & 0.4083 & 0.2523 \\ 0.2081 & 0.6145 & 0.1774 \\ 0.2477 & 0.3644 & 0.3879 \end{pmatrix}$$

Scenario 4

Markov Frequencies Matrixes for the Transition Occurrence from States i to j

$$n_{ij(ASIR)} = \begin{pmatrix} 0 & 2 & 1 & 1 \\ 1 & 1110 & 608 & 1 \\ 1 & 607 & 729 & 1 \\ 2 & 1 & 0 & 1 \end{pmatrix} \qquad P_{ij(ASIR)} = \begin{pmatrix} 0.0000 & 0.5000 & 0.25 & 0.25 \\ 0.0006 & 0.6453 & 0.3535 & 0.0006 \\ 0.0007 & 0.4540 & 0.5445 & 0.0007 \\ 0.5000 & 0.25 & 0.0000 & 0.25 \end{pmatrix}$$

$$n_{ij(\mathrm{DANGCEM}R)} = \begin{pmatrix} 25 & 97 & 38 & 28 \\ 108 & 745 & 200 & 90 \\ 19 & 216 & 88 & 33 \\ 36 & 84 & 31 & 28 \end{pmatrix} \qquad P_{ij(\mathrm{DANGCEM}R)} = \begin{pmatrix} 0.1330 & 0.5160 & 0.2021 & 0.1489 \\ 0.0945 & 0.6518 & 0.1750 & 0.0787 \\ 0.0534 & 0.6067 & 0.2472 & 0.0927 \\ 0.2011 & 0.4693 & 0.1732 & 0.1564 \end{pmatrix}$$

$$n_{ij(\mathrm{GTB}R)} = \begin{pmatrix} 75 & 85 & 78 & 50 \\ 121 & 684 & 492 & 69 \\ 47 & 503 & 409 & 110 \\ 45 & 94 & 90 & 113 \end{pmatrix} \qquad P_{ij(\mathrm{GTB}R)} = \begin{pmatrix} 0.2604 & 0.2951 & 0.2708 & 0.1736 \\ 0.0886 & 0.5007 & 0.3602 & 0.0505 \\ 0.0440 & 0.4705 & 0.3826 & 0.1029 \\ 0.1316 & 0.2749 & 0.2632 & 0.3304 \end{pmatrix}$$

Scenario 5

Markov Frequencies Matrixes for the Transition Occurrence from States i to j

$$n_{ij(ASIR)} = \begin{pmatrix} 318 & 143 & 171 & 135 \\ 155 & 220 & 207 & 109 \\ 166 & 203 & 264 & 212 \\ 128 & 125 & 203 & 307 \end{pmatrix} \qquad P_{ij(ASIR)} = \begin{pmatrix} 0.4146 & 0.1864 & 0.2229 & 0.1760 \\ 0.2243 & 0.3184 & 0.2996 & 0.1577 \\ 0.1964 & 0.2402 & 0.3124 & 0.2509 \\ 0.1680 & 0.1640 & 0.2664 & 0.4016 \end{pmatrix}$$

$$n_{ij(\mathrm{DANGCEM}R)} = \begin{pmatrix} 125 & 24 & 176 & 145 \\ 23 & 6 & 47 & 15 \\ 181 & 46 & 441 & 170 \\ 141 & 15 & 173 & 138 \end{pmatrix} \qquad P_{ij(\mathrm{DANGCEM}R)} = \begin{pmatrix} 0.2660 & 0.0511 & 0.3745 & 0.3085 \\ 0.2527 & 0.0659 & 0.5165 & 0.1648 \\ 0.2160 & 0.0549 & 0.5263 & 0.2029 \\ 0.3019 & 0.0321 & 0.3704 & 0.2955 \end{pmatrix}$$

$$n_{ij(\mathrm{GTB}R)} = \begin{pmatrix} 261 & 129 & 185 & 194 \\ 156 & 176 & 225 & 97 \\ 163 & 228 & 313 & 175 \\ 189 & 121 & 157 & 296 \end{pmatrix} \qquad P_{ij(\mathrm{GTB}R)} = \begin{pmatrix} 0.3394 & 0.1678 & 0.2406 & 0.2523 \\ 0.2385 & 0.2691 & 0.3440 & 0.1483 \\ 0.1853 & 0.2594 & 0.3561 & 0.1991 \\ 0.2477 & 0.1586 & 0.2058 & 0.3879 \end{pmatrix}$$

APPENDIX 4

Computation of Limiting Distribution

ASIR

Scenario One

2006-2018

$$P = \begin{pmatrix} 0.5734 & 0.4266 \\ 0.3871 & 0.6129 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0.4939 & 0.5061 \\ 0.4592 & 0.5408 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.4791 & 0.5209 \\ 0.4727 & 0.5273 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 0.4764 & 0.5236 \\ 0.4752 & 0.5248 \end{pmatrix}$$

$$P^5 = \begin{pmatrix} 0.4758 & 0.5242 \\ 0.4756 & 0.5244 \end{pmatrix}$$

$$P^6 = \begin{pmatrix} 0.4758 & 0.5242 \\ 0.4757 & 0.5243 \end{pmatrix}$$

$$P^7 = \begin{pmatrix} 0.4757 & 0.5243 \\ 0.4757 & 0.5243 \end{pmatrix}$$

$$P^8 = \begin{pmatrix} 0.4757 & 0.5243 \\ 0.4757 & 0.5243 \end{pmatrix}$$

$$P^9 = \begin{pmatrix} 0.4757 & 0.5243 \\ 0.4757 & 0.5243 \end{pmatrix}$$

$$P^{10} = \begin{pmatrix} 0.4757 & 0.5243 \\ 0.4757 & 0.5243 \end{pmatrix}$$

$$P^{12} = \begin{pmatrix} 0.4757 & 0.5243 \\ 0.4757 & 0.5243 \end{pmatrix}$$

$$P^{15} = \begin{pmatrix} 0.4757 & 0.5243 \\ 0.4757 & 0.5243 \end{pmatrix}$$

Scenario 2

2006-2018

$$P = \begin{pmatrix} 0.0000 & 0.75 & 0.25 \\ 0.0007 & 0.9987 & 0.0007 \\ 0.50 & 0.25 & 0.25 \end{pmatrix}$$

$$P^{2} = \begin{pmatrix} 0.1255 & 0.8115 & 0.0630 \\ 0.0010 & 0.9981 & 0.0010 \\ 0.1252 & 0.6872 & 0.1877 \end{pmatrix}$$

$$P^{3} = \begin{pmatrix} 0.0321 & 0.9204 & 0.0477 \\ 0.0012 & 0.9979 & 0.0012 \\ 0.0943 & 0.8271 & 0.0787 \end{pmatrix}$$

$$P^{4} = \begin{pmatrix} 0.0245 & 0.9552 & 0.0206 \\ 0.0013 & 0.9978 & 0.0013 \\ 0.0399 & 0.9164 & 0.0438 \end{pmatrix}$$

$$P^{5} = \begin{pmatrix} 0.0110 & 0.9774 & 0.0119 \\ 0.0014 & 0.9978 & 0.0014 \\ 0.0226 & 0.9561 & 0.0216 \end{pmatrix}$$

$$P^{6} = \begin{pmatrix} 0.0067 & 0.9874 & 0.0064 \\ 0.0014 & 0.9978 & 0.0014 \\ 0.0115 & 0.9772 & 0.0117 \end{pmatrix}$$

$$P^{7} = \begin{pmatrix} 0.0039 & 0.9927 & 0.0040 \\ 0.0014 & 0.9979 & 0.0014 \\ 0.0065 & 0.9875 & 0.0065 \end{pmatrix}$$

$$P^8 = \begin{pmatrix} 0.0027 & 0.9953 & 0.0027 \\ 0.0014 & 0.9980 & 0.0014 \\ 0.0039 & 0.9927 & 0.0039 \end{pmatrix}$$

$$P^9 = \begin{pmatrix} 0.0020 & 0.9967 & 0.0020 \\ 0.0014 & 0.9981 & 0.0014 \\ 0.0027 & 0.9953 & 0.0039 \end{pmatrix}$$

$$P^{10} = \begin{pmatrix} 0.0017 & 0.9974 & 0.0017 \\ 0.0014 & 0.9982 & 0.0014 \\ 0.0020 & 0.9967 & 0.0020 \end{pmatrix}$$

$$P^{12} = \begin{pmatrix} 0.0015 & 0.9981 & 0.0014 \\ 0.0014 & 0.9984 & 0.0014 \\ 0.0016 & 0.9978 & 0.0014 \end{pmatrix}$$

$$P^{15} = \begin{pmatrix} 0.0014 & 0.9985 & 0.0014 \\ 0.0014 & 0.9987 & 0.0014 \\ 0.0014 & 0.9984 & 0.0014 \end{pmatrix}$$

Scenario 3

2006-2018

$$P = \begin{pmatrix} 0.4146 & 0.4094 & 0.1760 \\ 0.2090 & 0.5820 & 0.2090 \\ 0.1680 & 0.4304 & 0.4016 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0.2870 & 0.4838 & 0.2292 \\ 0.2434 & 0.5142 & 0.2424 \\ 0.2271 & 0.4921 & 0.2808 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.2586 & 0.4977 & 0.2437 \\ 0.2491 & 0.5032 & 0.2476 \\ 0.2442 & 0.5002 & 0.2556 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 0.2522 & 0.5004 & 0.2474 \\ 0.2501 & 0.5015 & 0.2485 \\ 0.2487 & 0.5011 & 0.2502 \end{pmatrix}$$

$$P^5 = \begin{pmatrix} 0.2507 & 0.5010 & 0.2483 \\ 0.2502 & 0.5012 & 0.2486 \\ 0.2499 & 0.5011 & 0.2490 \end{pmatrix}$$

$$P^6 = \begin{pmatrix} 0.2504 & 0.5011 & 0.2486 \\ 0.2503 & 0.5011 & 0.2486 \\ 0.2502 & 0.5011 & 0.2587 \end{pmatrix}$$

$$P^7 = \begin{pmatrix} 0.2503 & 0.5011 & 0.2486 \\ 0.2503 & 0.5011 & 0.2486 \\ 0.2502 & 0.5011 & 0.2486 \end{pmatrix}$$

$$P^8 = \begin{pmatrix} 0.2503 & 0.5011 & 0.2486 \\ 0.2503 & 0.5011 & 0.2486 \\ 0.2503 & 0.5011 & 0.2486 \end{pmatrix}$$

$$P^9 = \begin{pmatrix} 0.2503 & 0.5011 & 0.2486 \\ 0.2503 & 0.5011 & 0.2486 \\ 0.2503 & 0.5011 & 0.2486 \end{pmatrix}$$

$$P^{10} = \begin{pmatrix} 0.2503 & 0.5011 & 0.2486 \\ 0.2503 & 0.5011 & 0.2486 \\ 0.2503 & 0.5011 & 0.2486 \end{pmatrix}$$

$$P^{12} = \begin{pmatrix} 0.2503 & 0.5011 & 0.2486 \\ 0.2503 & 0.5011 & 0.2486 \\ 0.2503 & 0.5011 & 0.2486 \end{pmatrix}$$

$$P^{15} = \begin{pmatrix} 0.2503 & 0.5011 & 0.2486 \\ 0.2503 & 0.5011 & 0.2486 \\ 0.2503 & 0.5011 & 0.2486 \end{pmatrix}$$

Scenario 4

2006-2018

$$P = \begin{pmatrix} 0.0000 & 0.5000 & 0.25 & 0.25 \\ 0.0006 & 0.6453 & 0.3535 & 0.0006 \\ 0.0007 & 0.4540 & 0.5445 & 0.0007 \\ 0.5000 & 0.25 & 0.0000 & 0.25 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0.1255 & 0.4986 & 0.3129 & 0.0630 \\ 0.0009 & 0.5774 & 0.4207 & 0.0009 \\ 0.0010 & 0.5407 & 0.4571 & 0.0010 \\ 0.1252 & 0.4738 & 0.2134 & 0.1876 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.0320 & 0.5423 & 0.3780 & 0.0476 \\ 0.0011 & 0.5643 & 0.4334 & 0.0011 \\ 0.0011 & 0.5572 & 0.4403 & 0.0011 \\ 0.0943 & 0.5121 & 0.3150 & 0.0786 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 0.0244 & 0.5495 & 0.4055 & 0.0205 \\ 0.0012 & 0.5617 & 0.4357 & 0.0012 \\ 0.0012 & 0.5603 & 0.4370 & 0.0012 \\ 0.0398 & 0.5403 & 0.3761 & 0.0438 \end{pmatrix}$$

$$P^5 = \begin{pmatrix} 0.0109 & 0.5560 & 0.4212 & 0.0118 \\ 0.0012 & 0.5612 & 0.4361 & 0.0012 \\ 0.0012 & 0.5609 & 0.4363 & 0.0012 \\ 0.0225 & 0.5502 & 0.4057 & 0.0215 \end{pmatrix}$$

$$P^6 = \begin{pmatrix} 0.0065 & 0.5584 & 0.4286 & 0.0063 \\ 0.0013 & 0.5611 & 0.4362 & 0.0013 \\ 0.0013 & 0.5610 & 0.4362 & 0.0013 \\ 0.0114 & 0.5559 & 0.4210 & 0.0116 \end{pmatrix}$$

$$P^7 = \begin{pmatrix} 0.0038 & 0.5598 & 0.4324 & 0.0038 \\ 0.0013 & 0.5610 & 0.4362 & 0.0013 \\ 0.0013 & 0.5610 & 0.4361 & 0.0013 \\ 0.0114 & 0.5584 & 0.4286 & 0.0064 \end{pmatrix}$$

$$P^8 = \begin{pmatrix} 0.0026 & 0.5604 & 0.4343 & 0.0025 \\ 0.0013 & 0.5610 & 0.4361 & 0.0013 \\ 0.0013 & 0.5609 & 0.4361 & 0.0013 \\ 0.0038 & 0.5597 & 0.4324 & 0.0038 \end{pmatrix}$$

$$P^9 = \begin{pmatrix} 0.0019 & 0.5607 & 0.4352 & 0.0019 \\ 0.0013 & 0.5610 & 0.4361 & 0.0013 \\ 0.0013 & 0.5609 & 0.4361 & 0.0013 \\ 0.0026 & 0.5604 & 0.4343 & 0.0026 \end{pmatrix}$$

$$P^{10} = \begin{pmatrix} 0.0016 & 0.5608 & 0.4356 & 0.0016 \\ 0.0013 & 0.5610 & 0.4361 & 0.0013 \\ 0.0013 & 0.5609 & 0.4360 & 0.0013 \\ 0.0019 & 0.5607 & 0.4352 & 0.0019 \end{pmatrix}$$

$$P^{12} = \begin{pmatrix} 0.0014 & 0.5609 & 0.4360 & 0.0014 \\ 0.0013 & 0.5609 & 0.4361 & 0.0013 \\ 0.0013 & 0.5608 & 0.4360 & 0.0013 \\ 0.0014 & 0.5609 & 0.4359 & 0.0014 \end{pmatrix}$$

$$P^{15} = \begin{pmatrix} 0.0013 & 0.5609 & 0.4360 & 0.0013 \\ 0.0013 & 0.5608 & 0.4360 & 0.0013 \\ 0.0013 & 0.5608 & 0.4359 & 0.0013 \\ 0.0013 & 0.5609 & 0.4360 & 0.0013 \end{pmatrix}$$

Scenario 5

2001-2018

$$P = \begin{pmatrix} 0.4146 & 0.1864 & 0.2229 & 0.1760 \\ 0.2243 & 0.3184 & 0.2996 & 0.1577 \\ 0.1964 & 0.2402 & 0.3124 & 0.2509 \\ 0.1680 & 0.1640 & 0.2664 & 0.4016 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0.2870 & 0.2190 & 0.2648 & 0.2290 \\ 0.2497 & 0.2410 & 0.2810 & 0.2282 \\ 0.2388 & 0.2293 & 0.2802 & 0.2516 \\ 0.2262 & 0.2134 & 0.2768 & 0.2836 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.2586 & 0.2244 & 0.2733 & 0.2435 \\ 0.2511 & 0.2282 & 0.2764 & 0.2441 \\ 0.2477 & 0.2261 & 0.2765 & 0.2495 \\ 0.2437 & 0.2231 & 0.2764 & 0.2568 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 0.2521 & 0.2252 & 0.2751 & 0.2472 \\ 0.2506 & 0.2259 & 0.2757 & 0.2476 \\ 0.2496 & 0.2255 & 0.2758 & 0.2488 \\ 0.2485 & 0.2249 & 0.2759 & 0.2505 \end{pmatrix}$$

$$P^5 = \begin{pmatrix} 0.2506 & 0.2253 & 0.2755 & 0.2482 \\ 0.2503 & 0.2255 & 0.2756 & 0.2483 \\ 0.2500 & 0.2254 & 0.2756 & 0.2486 \\ 0.2498 & 0.2253 & 0.2757 & 0.2490 \end{pmatrix}$$

$$P^6 = \begin{pmatrix} 0.2503 & 0.2253 & 0.2756 & 0.2485 \\ 0.2502 & 0.2254 & 0.2756 & 0.2485 \\ 0.2501 & 0.2254 & 0.2756 & 0.2486 \\ 0.2501 & 0.2254 & 0.2756 & 0.2587 \end{pmatrix}$$

$$P^7 = \begin{pmatrix} 0.2502 & 0.2253 & 0.2756 & 0.2485 \\ 0.2502 & 0.2254 & 0.2756 & 0.2485 \\ 0.2501 & 0.2253 & 0.2756 & 0.2485 \\ 0.2501 & 0.2254 & 0.2756 & 0.2486 \end{pmatrix}$$

$$P^8 = \begin{pmatrix} 0.2501 & 0.2253 & 0.2756 & 0.2485 \\ 0.2502 & 0.2253 & 0.2756 & 0.2485 \\ 0.2501 & 0.2253 & 0.2756 & 0.2485 \\ 0.2501 & 0.2254 & 0.2756 & 0.2485 \end{pmatrix}$$

$$P^9 = \begin{pmatrix} 0.2501 & 0.2253 & 0.2755 & 0.2485 \\ 0.2501 & 0.2253 & 0.2756 & 0.2485 \\ 0.2501 & 0.2253 & 0.2756 & 0.2485 \\ 0.2501 & 0.2253 & 0.2756 & 0.2485 \end{pmatrix}$$

$$P^{10} = \begin{pmatrix} 0.2501 & 0.2253 & 0.2755 & 0.2485 \\ 0.2501 & 0.2253 & 0.2756 & 0.2485 \\ 0.2501 & 0.2253 & 0.2755 & 0.2485 \\ 0.2501 & 0.2253 & 0.2756 & 0.2485 \end{pmatrix}$$

$$P^{12} = \begin{pmatrix} 0.2501 & 0.2253 & 0.2755 & 0.2485 \\ 0.2501 & 0.2253 & 0.2755 & 0.2485 \\ 0.2501 & 0.2253 & 0.2755 & 0.2485 \\ 0.2501 & 0.2253 & 0.2755 & 0.2485 \end{pmatrix}$$

$$P^{15} = \begin{pmatrix} 0.2500 & 0.2252 & 0.2755 & 0.2484 \\ 0.2501 & 0.2253 & 0.2755 & 0.2484 \\ 0.2500 & 0.2252 & 0.2755 & 0.2484 \\ 0.2501 & 0.2253 & 0.2755 & 0.2484 \end{pmatrix}$$

DANGCEMR 2010-2018

Scenario 1

$$P = \begin{pmatrix} 0.3173 & 0.6827 \\ 0.2935 & 0.7065 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0.3011 & 0.6989 \\ 0.3005 & 0.6995 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.3007 & 0.6993 \\ 0.3007 & 0.6993 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 0.3007 & 0.6993 \\ 0.3007 & 0.6993 \end{pmatrix}$$

$$P^5 = \begin{pmatrix} 0.3007 & 0.6993 \\ 0.3007 & 0.6993 \end{pmatrix}$$

$$P^6 = \begin{pmatrix} 0.3007 & 0.6993 \\ 0.3007 & 0.6993 \end{pmatrix}$$

$$P^7 = \begin{pmatrix} 0.3007 & 0.6993 \\ 0.3007 & 0.6993 \end{pmatrix}$$

$$P^8 = \begin{pmatrix} 0.3007 & 0.6993 \\ 0.3007 & 0.6993 \end{pmatrix}$$

$$P^9 = \begin{pmatrix} 0.3007 & 0.6993 \\ 0.3007 & 0.6993 \end{pmatrix}$$

$$P^{10} = \begin{pmatrix} 0.3007 & 0.6993 \\ 0.3007 & 0.6993 \end{pmatrix}$$

$$P^{12} = \begin{pmatrix} 0.3007 & 0.6993 \\ 0.3007 & 0.6993 \end{pmatrix}$$

$$P^{15} = \begin{pmatrix} 0.3007 & 0.6993 \\ 0.3007 & 0.6993 \end{pmatrix}$$

Scenario 2

2010-2018

$$P = \begin{pmatrix} 0.1330 & 0.7181 & 0.1489 \\ 0.0847 & 0.8332 & 0.0821 \\ 0.2011 & 0.6425 & 0.1564 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0.1085 & 0.7895 & 0.1020 \\ 0.0983 & 0.8078 & 0.0939 \\ 0.1126 & 0.7802 & 0.1072 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.1018 & 0.8013 & 0.0969 \\ 0.1004 & 0.8040 & 0.0956 \\ 0.1026 & 0.7998 & 0.0976 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 0.1009 & 0.8030 & 0.0961 \\ 0.1007 & 0.8034 & 0.0959 \\ 0.1010 & 0.8028 & 0.0962 \end{pmatrix}$$

$$P^5 = \begin{pmatrix} 0.1008 & 0.8033 & 0.0960 \\ 0.1007 & 0.8033 & 0.0960 \\ 0.1008 & 0.8032 & 0.0960 \end{pmatrix}$$

$$P^6 = \begin{pmatrix} 0.1007 & 0.8033 & 0.096 \\ 0.1007 & 0.8033 & 0.096 \\ 0.1007 & 0.8033 & 0.096 \end{pmatrix}$$

$$P^7 = \begin{pmatrix} 0.1007 & 0.8033 & 0.096 \\ 0.1007 & 0.8033 & 0.096 \\ 0.1007 & 0.8033 & 0.096 \end{pmatrix}$$

$$P^8 = \begin{pmatrix} 0.1007 & 0.8033 & 0.096 \\ 0.1007 & 0.8033 & 0.096 \\ 0.1007 & 0.8033 & 0.096 \end{pmatrix}$$

$$P^9 = \begin{pmatrix} 0.1007 & 0.8033 & 0.096 \\ 0.1007 & 0.8033 & 0.096 \\ 0.1007 & 0.8033 & 0.096 \end{pmatrix}$$

$$P^{10} = \begin{pmatrix} 0.1007 & 0.8033 & 0.096 \\ 0.1007 & 0.8033 & 0.096 \\ 0.1007 & 0.8033 & 0.096 \end{pmatrix}$$

$$P^{12} = \begin{pmatrix} 0.1007 & 0.8033 & 0.096 \\ 0.1007 & 0.8033 & 0.096 \\ 0.1007 & 0.8033 & 0.096 \end{pmatrix}$$

$$P^{15} = \begin{pmatrix} 0.1007 & 0.8033 & 0.096 \\ 0.1007 & 0.8033 & 0.096 \\ 0.1007 & 0.8033 & 0.096 \end{pmatrix}$$

Scenario 3

2010-2018

$$P = \begin{pmatrix} 0.2660 & 0.4255 & 0.3085 \\ 0.2196 & 0.5813 & 0.1991 \\ 0.3019 & 0.4026 & 0.2955 \end{pmatrix}$$

$$P^{2} = \begin{pmatrix} 0.2573 & 0.4847 & 0.2579 \\ 0.2462 & 0.5115 & 0.2423 \\ 0.2579 & 0.4815 & 0.2606 \end{pmatrix}$$

$$P^{3} = \begin{pmatrix} 0.2528 & 0.4951 & 0.2521 \\ 0.2510 & 0.4996 & 0.2494 \\ 0.2530 & 0.4945 & 0.2524 \end{pmatrix}$$

$$P^{4} = \begin{pmatrix} 0.2521 & 0.4969 & 0.2511 \\ 0.2518 & 0.4976 & 0.2506 \\ 0.2521 & 0.4968 & 0.2511 \end{pmatrix}$$

$$P^5 = \begin{pmatrix} 0.2520 & 0.4972 & 0.2509 \\ 0.2519 & 0.4973 & 0.2508 \\ 0.2520 & 0.4971 & 0.2509 \end{pmatrix}$$

$$P^6 = \begin{pmatrix} 0.2519 & 0.4972 & 0.2508 \\ 0.2519 & 0.4972 & 0.2508 \\ 0.2519 & 0.4972 & 0.2509 \end{pmatrix}$$

$$P^7 = \begin{pmatrix} 0.2519 & 0.4972 & 0.2508 \\ 0.2519 & 0.4972 & 0.2508 \\ 0.2519 & 0.4972 & 0.2508 \end{pmatrix}$$

$$P^8 = \begin{pmatrix} 0.2519 & 0.4972 & 0.2508 \\ 0.2519 & 0.4972 & 0.2508 \\ 0.2519 & 0.4972 & 0.2508 \end{pmatrix}$$

$$P^9 = \begin{pmatrix} 0.2519 & 0.4972 & 0.2508 \\ 0.2519 & 0.4972 & 0.2508 \\ 0.2519 & 0.4972 & 0.2508 \end{pmatrix}$$

$$P^{10} = \begin{pmatrix} 0.2519 & 0.4972 & 0.2508 \\ 0.2519 & 0.4972 & 0.2508 \\ 0.2519 & 0.4972 & 0.2508 \end{pmatrix}$$

$$P^{12} = \begin{pmatrix} 0.2519 & 0.4972 & 0.2508 \\ 0.2519 & 0.4972 & 0.2508 \\ 0.2519 & 0.4972 & 0.2508 \end{pmatrix}$$

$$P^{15} = \begin{pmatrix} 0.2519 & 0.4972 & 0.2508 \\ 0.2519 & 0.4972 & 0.2508 \\ 0.2519 & 0.4972 & 0.2508 \end{pmatrix}$$

Scenario 4

2010-2018

$$P = \begin{pmatrix} 0.1330 & 0.5160 & 0.2021 & 0.1489 \\ 0.0945 & 0.6518 & 0.1750 & 0.0787 \\ 0.0534 & 0.6067 & 0.2472 & 0.0927 \\ 0.2011 & 0.4693 & 0.1732 & 0.1564 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0.1072 & 0.5974 & 0.1929 & 0.1024 \\ 0.0993 & 0.6167 & 0.1901 & 0.0939 \\ 0.0963 & 0.6165 & 0.1941 & 0.0931 \\ 0.1118 & 0.5881 & 0.1927 & 0.1074 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.1016 & 0.6098 & 0.1916 & 0.0969 \\ 0.1005 & 0.6126 & 0.1912 & 0.0956 \\ 0.1002 & 0.6130 & 0.1915 & 0.0954 \\ 0.1023 & 0.6083 & 0.1917 & 0.0976 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 0.1009 & 0.6117 & 0.1914 & 0.0960 \\ 0.1007 & 0.6121 & 0.1914 & 0.0959 \\ 0.1007 & 0.6122 & 0.1914 & 0.0958 \\ 0.1010 & 0.6114 & 0.1914 & 0.0962 \end{pmatrix}$$

$$P^5 = \begin{pmatrix} 0.1008 & 0.6119 & 0.1914 & 0.0959 \\ 0.1007 & 0.6120 & 0.1914 & 0.0959 \\ 0.1007 & 0.6120 & 0.1914 & 0.0959 \\ 0.1008 & 0.6119 & 0.1914 & 0.0959 \end{pmatrix}$$

$$P^6 = \begin{pmatrix} 0.1007 & 0.6120 & 0.1914 & 0.0959 \\ 0.1007 & 0.6120 & 0.1914 & 0.0959 \\ 0.1007 & 0.6120 & 0.1914 & 0.0959 \\ 0.1007 & 0.6120 & 0.1914 & 0.0959 \end{pmatrix}$$

$$P^7 = \begin{pmatrix} 0.1007 & 0.6120 & 0.1914 & 0.0959 \\ 0.1007 & 0.6120 & 0.1914 & 0.0959 \\ 0.1007 & 0.6120 & 0.1914 & 0.0959 \\ 0.1007 & 0.6120 & 0.1914 & 0.0959 \end{pmatrix}$$

$$P^{8}=\begin{pmatrix}0.1007 & 0.6120 & 0.1914 & 0.0959\\ 0.1007 & 0.6120 & 0.1914 & 0.0959\\ 0.1007 & 0.6120 & 0.1914 & 0.0959\\ 0.1007 & 0.6120 & 0.1914 & 0.0959\end{pmatrix}$$

$$P^{9}=\begin{pmatrix}0.1007 & 0.6120 & 0.1914 & 0.0959\\ 0.1007 & 0.6120 & 0.1914 & 0.0959\\ 0.1007 & 0.6120 & 0.1914 & 0.0959\\ 0.1007 & 0.6120 & 0.1914 & 0.0959\end{pmatrix}$$

$$P^{10}=\begin{pmatrix}0.1007 & 0.6120 & 0.1914 & 0.0959\\ 0.1007 & 0.6120 & 0.1914 & 0.0959\\ 0.1007 & 0.6120 & 0.1914 & 0.0959\\ 0.1007 & 0.6120 & 0.1914 & 0.0959\end{pmatrix}$$

$$P^{12}=\begin{pmatrix}0.1007 & 0.6120 & 0.1914 & 0.0959\\ 0.1007 & 0.6120 & 0.1914 & 0.0959\\ 0.1007 & 0.6120 & 0.1914 & 0.0959\\ 0.1007 & 0.6120 & 0.1914 & 0.0959\end{pmatrix}$$

$$P^{15}=\begin{pmatrix}0.1007 & 0.6120 & 0.1914 & 0.0959\\ 0.1007 & 0.6120 & 0.1914 & 0.0959\\ 0.1007 & 0.6120 & 0.1914 & 0.0959\\ 0.1007 & 0.6120 & 0.1914 & 0.0959\end{pmatrix}$$

Scenario 5

2010-2018

$$P=\begin{pmatrix}0.2660 & 0.0511 & 0.3745 & 0.3085\\ 0.2527 & 0.0659 & 0.5165 & 0.1648\\ 0.2160 & 0.0549 & 0.5263 & 0.2029\\ 0.3019 & 0.0321 & 0.3704 & 0.2955\end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0.2577 & 0.0474 & 0.4374 & 0.2576 \\ 0.2452 & 0.0509 & 0.4615 & 0.2423 \\ 0.2463 & 0.0501 & 0.4614 & 0.2424 \\ 0.2576 & 0.0474 & 0.4340 & 0.2609 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.2528 & 0.0486 & 0.4466 & 0.2522 \\ 0.2509 & 0.0490 & 0.4508 & 0.2493 \\ 0.2510 & 0.0490 & 0.4507 & 0.2495 \\ 0.2530 & 0.0485 & 0.4460 & 0.2524 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 0.2521 & 0.0487 & 0.4482 & 0.2511 \\ 0.2518 & 0.0488 & 0.4489 & 0.2506 \\ 0.2518 & 0.0488 & 0.4489 & 0.2507 \\ 0.2521 & 0.0487 & 0.4480 & 0.2511 \end{pmatrix}$$

$$P^5 = \begin{pmatrix} 0.2520 & 0.0488 & 0.4485 & 0.2510 \\ 0.2519 & 0.0488 & 0.4486 & 0.2508 \\ 0.2520 & 0.0488 & 0.4486 & 0.2509 \\ 0.2520 & 0.0488 & 0.4484 & 0.2509 \end{pmatrix}$$

$$P^6 = \begin{pmatrix} 0.2520 & 0.0488 & 0.4486 & 0.2509 \\ 0.2519 & 0.0488 & 0.4485 & 0.2509 \\ 0.2520 & 0.0488 & 0.4486 & 0.2509 \\ 0.2520 & 0.0488 & 0.4485 & 0.2509 \end{pmatrix}$$

$$P^7 = \begin{pmatrix} 0.2520 & 0.0488 & 0.4486 & 0.2509 \\ 0.2520 & 0.0488 & 0.4485 & 0.2509 \\ 0.2520 & 0.0488 & 0.4486 & 0.2510 \\ 0.2520 & 0.0488 & 0.4485 & 0.2509 \end{pmatrix}$$

$$P^8 = \begin{pmatrix} 0.2520 & 0.0488 & 0.4486 & 0.2510 \\ 0.2520 & 0.0488 & 0.4485 & 0.2509 \\ 0.2520 & 0.0488 & 0.4486 & 0.2510 \\ 0.2520 & 0.0488 & 0.4485 & 0.2509 \end{pmatrix}$$

$$P^9 = \begin{pmatrix} 0.2520 & 0.0488 & 0.4486 & 0.2510 \\ 0.2520 & 0.0488 & 0.4486 & 0.2509 \\ 0.2520 & 0.0488 & 0.4486 & 0.2510 \\ 0.2520 & 0.0488 & 0.4485 & 0.2509 \end{pmatrix}$$

$$P^{10} = \begin{pmatrix} 0.2520 & 0.0488 & 0.4487 & 0.2510 \\ 0.2520 & 0.0488 & 0.4486 & 0.2509 \\ 0.2520 & 0.0488 & 0.4487 & 0.2510 \\ 0.2520 & 0.0488 & 0.4486 & 0.2509 \end{pmatrix}$$

$$P^{12} = \begin{pmatrix} 0.2521 & 0.0488 & 0.4487 & 0.2510 \\ 0.2520 & 0.0488 & 0.4486 & 0.2510 \\ 0.2521 & 0.0488 & 0.4487 & 0.2510 \\ 0.2520 & 0.0488 & 0.4486 & 0.2509 \end{pmatrix}$$

$$P^{15} = \begin{pmatrix} 0.2521 & 0.0488 & 0.4487 & 0.2510 \\ 0.2520 & 0.0488 & 0.4487 & 0.2510 \\ 0.2521 & 0.0488 & 0.4488 & 0.2510 \\ 0.2520 & 0.0488 & 0.4487 & 0.2510 \end{pmatrix}$$

GTBR

Scenario One

2006-2018

$$P = \begin{pmatrix} 0.5074 & 0.4926 \\ 0.4269 & 0.5731 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0.4677 & 0.5323 \\ 0.4613 & 0.5387 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.4646 & 0.5354 \\ 0.4640 & 0.5360 \end{pmatrix}$$

$$P^{4} = \begin{pmatrix} 0.4643 & 0.5357 \\ 0.4643 & 0.5357 \end{pmatrix}$$

$$P^{5} = \begin{pmatrix} 0.4643 & 0.5357 \\ 0.4643 & 0.5357 \end{pmatrix}$$

$$P^{6} = \begin{pmatrix} 0.4643 & 0.5357 \\ 0.4643 & 0.5357 \end{pmatrix}$$

$$P^{7} = \begin{pmatrix} 0.4643 & 0.5357 \\ 0.4643 & 0.5357 \end{pmatrix}$$

$$P^{8} = \begin{pmatrix} 0.4643 & 0.5357 \\ 0.4643 & 0.5357 \end{pmatrix}$$

$$P^{9} = \begin{pmatrix} 0.4643 & 0.5357 \\ 0.4643 & 0.5357 \end{pmatrix}$$

$$P^{10} = \begin{pmatrix} 0.4643 & 0.5357 \\ 0.4643 & 0.5357 \end{pmatrix}$$

$$P^{12} = \begin{pmatrix} 0.4643 & 0.5357 \\ 0.4643 & 0.5357 \end{pmatrix}$$

$$P^{15} = \begin{pmatrix} 0.4643 & 0.5357 \\ 0.4643 & 0.5357 \end{pmatrix}$$

Scenario 2

2006-2018

$$P = \begin{pmatrix} 0.2604 & 0.5660 & 0.1736 \\ 0.0690 & 0.8575 & 0.0735 \\ 0.1316 & 0.5380 & 0.3304 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0.1297 & 0.7261 & 0.1442 \\ 0.0868 & 0.8139 & 0.0993 \\ 0.1149 & 0.7136 & 0.1716 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.1029 & 0.7736 & 0.1235 \\ 0.0918 & 0.8005 & 0.1077 \\ 0.1017 & 0.7692 & 0.1291 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 0.0964 & 0.7881 & 0.1155 \\ 0.0933 & 0.7963 & 0.1104 \\ 0.0966 & 0.7866 & 0.1168 \end{pmatrix}$$

$$P^5 = \begin{pmatrix} 0.0947 & 0.7925 & 0.1128 \\ 0.0938 & 0.7950 & 0.1112 \\ 0.0948 & 0.7920 & 0.1132 \end{pmatrix}$$

$$P^6 = \begin{pmatrix} 0.0942 & 0.7938 & 0.1120 \\ 0.0939 & 0.7946 & 0.1115 \\ 0.0942 & 0.7937 & 0.1121 \end{pmatrix}$$

$$P^7 = \begin{pmatrix} 0.0940 & 0.7943 & 0.1117 \\ 0.0940 & 0.7945 & 0.1115 \\ 0.0941 & 0.7942 & 0.1117 \end{pmatrix}$$

$$P^8 = \begin{pmatrix} 0.0940 & 0.7944 & 0.1116 \\ 0.0940 & 0.7945 & 0.1116 \\ 0.0940 & 0.7944 & 0.1116 \end{pmatrix}$$

$$P^{10} = \begin{pmatrix} 0.0940 & 0.7945 & 0.1116 \\ 0.0940 & 0.7945 & 0.1116 \\ 0.0940 & 0.7945 & 0.1116 \end{pmatrix}$$

$$P^{12} = \begin{pmatrix} 0.0940 & 0.7945 & 0.1116 \\ 0.0940 & 0.7945 & 0.1116 \\ 0.0940 & 0.7945 & 0.1116 \end{pmatrix}$$

$$P^{10} = \begin{pmatrix} 0.0940 & 0.7945 & 0.1116 \\ 0.0940 & 0.7945 & 0.1116 \\ 0.0940 & 0.7945 & 0.1116 \end{pmatrix}$$

Scenario 3

2006-2018

$$P = \begin{pmatrix} 0.3394 & 0.4083 & 0.2523 \\ 0.2081 & 0.6145 & 0.1774 \\ 0.2471 & 0.3644 & 0.3879 \end{pmatrix}$$

$$P^{2} = \begin{pmatrix} 0.2627 & 0.4814 & 0.2559 \\ 0.2424 & 0.5272 & 0.2303 \\ 0.2560 & 0.4664 & 0.2776 \end{pmatrix}$$

$$P^{3} = \begin{pmatrix} 0.2527 & 0.4963 & 0.2509 \\ 0.2491 & 0.5069 & 0.2440 \\ 0.2527 & 0.4923 & 0.2550 \end{pmatrix}$$

$$P^{4} = \begin{pmatrix} 0.2512 & 0.4996 & 0.2492 \\ 0.2505 & 0.5021 & 0.2474 \\ 0.2514 & 0.4986 & 0.2500 \end{pmatrix}$$

$$P^{5} = \begin{pmatrix} 0.2510 & 0.5004 & 0.2487 \\ 0.2508 & 0.5010 & 0.2482 \\ 0.2510 & 0.5001 & 0.2489 \end{pmatrix}$$

$$P^6 = \begin{pmatrix} 0.2509 & 0.5006 & 0.2485 \\ 0.2509 & 0.5007 & 0.2484 \\ 0.2509 & 0.5005 & 0.2486 \end{pmatrix}$$

$$P^7 = \begin{pmatrix} 0.2509 & 0.5006 & 0.2485 \\ 0.2509 & 0.5006 & 0.2485 \\ 0.2509 & 0.5006 & 0.2485 \end{pmatrix}$$

$$P^8 = \begin{pmatrix} 0.2509 & 0.5006 & 0.2485 \\ 0.2509 & 0.5006 & 0.2485 \\ 0.2509 & 0.5006 & 0.2485 \end{pmatrix}$$

$$P^9 = \begin{pmatrix} 0.2509 & 0.5006 & 0.2485 \\ 0.2509 & 0.5006 & 0.2485 \\ 0.2509 & 0.5006 & 0.2485 \end{pmatrix}$$

$$P^{10} = \begin{pmatrix} 0.2509 & 0.5006 & 0.2485 \\ 0.2509 & 0.5006 & 0.2485 \\ 0.2509 & 0.5006 & 0.2485 \end{pmatrix}$$

$$P^{12} = \begin{pmatrix} 0.2509 & 0.5006 & 0.2485 \\ 0.2509 & 0.5006 & 0.2485 \\ 0.2509 & 0.5006 & 0.2485 \end{pmatrix}$$

$$P^{15} = \begin{pmatrix} 0.2509 & 0.5006 & 0.2485 \\ 0.2509 & 0.5006 & 0.2485 \\ 0.2509 & 0.5006 & 0.2485 \end{pmatrix}$$

Scenario 4

2006-2018

$$P = \begin{pmatrix} 0.2604 & 0.2951 & 0.2708 & 0.1736 \\ 0.0886 & 0.5007 & 0.3602 & 0.0505 \\ 0.0440 & 0.4705 & 0.3826 & 0.1029 \\ 0.1316 & 0.2749 & 0.2632 & 0.3304 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0.1287 & 0.3997 & 0.3261 & 0.1453 \\ 0.0899 & 0.4602 & 0.3554 & 0.0944 \\ 0.0835 & 0.4569 & 0.3549 & 0.1048 \\ 0.1137 & 0.3911 & 0.3223 & 0.1730 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.1024 & 0.4315 & 0.3419 & 0.1241 \\ 0.0923 & 0.4502 & 0.3510 & 0.1066 \\ 0.0916 & 0.4492 & 0.3505 & 0.1087 \\ 0.1012 & 0.4286 & 0.3405 & 0.1298 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 0.0963 & 0.4412 & 0.3466 & 0.1158 \\ 0.0934 & 0.4471 & 0.3495 & 0.1101 \\ 0.0934 & 0.4467 & 0.3493 & 0.1106 \\ 0.0964 & 0.4404 & 0.3462 & 0.1171 \end{pmatrix}$$

$$P^5 = \begin{pmatrix} 0.0946 & 0.4442 & 0.3481 & 0.1129 \\ 0.0938 & 0.4461 & 0.3490 & 0.1111 \\ 0.0938 & 0.4460 & 0.3490 & 0.1112 \\ 0.0948 & 0.4440 & 0.3480 & 0.1133 \end{pmatrix}$$

$$P^6 = \begin{pmatrix} 0.0942 & 0.4452 & 0.3485 & 0.1120 \\ 0.0939 & 0.4458 & 0.3489 & 0.1114 \\ 0.0939 & 0.4458 & 0.3488 & 0.1115 \\ 0.0942 & 0.4452 & 0.3486 & 0.1121 \end{pmatrix}$$

$$P^7 = \begin{pmatrix} 0.0940 & 0.4455 & 0.3487 & 0.1117 \\ 0.0940 & 0.4457 & 0.3488 & 0.1115 \\ 0.0940 & 0.4457 & 0.3488 & 0.1115 \\ 0.0941 & 0.4455 & 0.3488 & 0.1118 \end{pmatrix}$$

$$P^8 = \begin{pmatrix} 0.0940 & 0.4456 & 0.3487 & 0.1116 \\ 0.0940 & 0.4457 & 0.3488 & 0.1116 \\ 0.0940 & 0.4457 & 0.3488 & 0.1116 \\ 0.0940 & 0.4457 & 0.3488 & 0.1116 \end{pmatrix}$$

$$P^9 = \begin{pmatrix} 0.0940 & 0.4456 & 0.3487 & 0.1116 \\ 0.0940 & 0.4457 & 0.3488 & 0.1116 \\ 0.0940 & 0.4457 & 0.3488 & 0.1116 \\ 0.0940 & 0.4457 & 0.3488 & 0.1116 \end{pmatrix}$$

$$P^{10} = \begin{pmatrix} 0.0940 & 0.4456 & 0.3488 & 0.1116 \\ 0.0940 & 0.4457 & 0.3488 & 0.1116 \\ 0.0940 & 0.4457 & 0.3488 & 0.1116 \\ 0.0940 & 0.4457 & 0.3488 & 0.1116 \end{pmatrix}$$

$$P^{12} = \begin{pmatrix} 0.0940 & 0.4456 & 0.3488 & 0.1116 \\ 0.0940 & 0.4457 & 0.3488 & 0.1116 \\ 0.0940 & 0.4457 & 0.3488 & 0.1116 \\ 0.0940 & 0.4457 & 0.3488 & 0.1116 \end{pmatrix}$$

$$P^{15} = \begin{pmatrix} 0.0940 & 0.4456 & 0.3488 & 0.1116 \\ 0.0940 & 0.4457 & 0.3488 & 0.1116 \\ 0.0940 & 0.4457 & 0.3488 & 0.1116 \\ 0.0940 & 0.4457 & 0.3488 & 0.1116 \end{pmatrix}$$

Scenario 5

2001-2018

$$P = \begin{pmatrix} 0.3394 & 0.1678 & 0.2406 & 0.2523 \\ 0.2385 & 0.2691 & 0.3440 & 0.1483 \\ 0.1854 & 0.2594 & 0.3561 & 0.1991 \\ 0.2477 & 0.1586 & 0.2058 & 0.3879 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0.2623 & 0.2045 & 0.2770 & 0.2563 \\ 0.2456 & 0.2252 & 0.3030 & 0.2261 \\ 0.2401 & 0.2249 & 0.3016 & 0.2334 \\ 0.2561 & 0.1991 & 0.2673 & 0.2775 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.2526 & 0.2116 & 0.2848 & 0.2511 \\ 0.2493 & 0.2163 & 0.2910 & 0.2434 \\ 0.2489 & 0.2161 & 0.2906 & 0.2445 \\ 0.2527 & 0.2099 & 0.2824 & 0.2550 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 0.2512 & 0.2130 & 0.2867 & 0.2492 \\ 0.2504 & 0.2141 & 0.2881 & 0.2473 \\ 0.2504 & 0.2141 & 0.2880 & 0.2475 \\ 0.2514 & 0.2126 & 0.2861 & 0.2500 \end{pmatrix}$$

$$P^5 = \begin{pmatrix} 0.2509 & 0.2134 & 0.2871 & 0.2487 \\ 0.2507 & 0.2136 & 0.2874 & 0.2482 \\ 0.2508 & 0.2136 & 0.2874 & 0.2483 \\ 0.2510 & 0.2132 & 0.2869 & 0.2489 \end{pmatrix}$$

$$P^6 = \begin{pmatrix} 0.2509 & 0.2134 & 0.2872 & 0.2486 \\ 0.2508 & 0.2135 & 0.2872 & 0.2484 \\ 0.2508 & 0.2135 & 0.2872 & 0.2485 \\ 0.2509 & 0.2134 & 0.2871 & 0.2486 \end{pmatrix}$$

$$P^7 = \begin{pmatrix} 0.2509 & 0.2135 & 0.2872 & 0.2486 \\ 0.2508 & 0.2134 & 0.2872 & 0.2485 \\ 0.2508 & 0.2135 & 0.2872 & 0.2485 \\ 0.2509 & 0.2134 & 0.2872 & 0.2485 \end{pmatrix}$$

$$P^8 = \begin{pmatrix} 0.2509 & 0.2135 & 0.2872 & 0.2486 \\ 0.2508 & 0.2134 & 0.2872 & 0.2485 \\ 0.2508 & 0.2134 & 0.2872 & 0.2485 \\ 0.2509 & 0.2134 & 0.2872 & 0.2485 \end{pmatrix}$$

$$P^9 = \begin{pmatrix} 0.2509 & 0.2135 & 0.2872 & 0.2486 \\ 0.2508 & 0.2134 & 0.2872 & 0.2485 \\ 0.2508 & 0.2134 & 0.2872 & 0.2485 \\ 0.2509 & 0.2134 & 0.2872 & 0.2485 \end{pmatrix}$$

$$P^{10} = \begin{pmatrix} 0.2509 & 0.2135 & 0.2872 & 0.2486 \\ 0.2508 & 0.2134 & 0.2872 & 0.2485 \\ 0.2509 & 0.2134 & 0.2872 & 0.2485 \\ 0.2509 & 0.2134 & 0.2872 & 0.2485 \end{pmatrix}$$

$$P^{12} = \begin{pmatrix} 0.2509 & 0.2135 & 0.2872 & 0.2486 \\ 0.2508 & 0.2134 & 0.2872 & 0.2485 \\ 0.2509 & 0.2134 & 0.2872 & 0.2485 \\ 0.2509 & 0.2135 & 0.2872 & 0.2485 \end{pmatrix}$$

$$P^{15} = \begin{pmatrix} 0.2509 & 0.2135 & 0.2872 & 0.2486 \\ 0.2508 & 0.2134 & 0.2872 & 0.2485 \\ 0.2509 & 0.2134 & 0.2872 & 0.2485 \\ 0.2509 & 0.2135 & 0.2872 & 0.2485 \end{pmatrix}$$